LBM FOR TRANSIENT COUPLED RB CONVECTION WITH RADIATION

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Abstract - in this paper transient Rayleigh-Bénard convection and volumetric radiation are investigated by means of the lattice Boltzmann method (LBM). First, Rayleigh-Bénard convection is solved using LBM and then are compared with those available in the literature. Then the combined transient case, Rayleigh-Bénard convection-radiation in participating media is extended, where LBM, is used, both to calculate the volumetric radiative information needed for the energy equation, which is solved using the LBM. In all cases, good agreement has been obtained. The recent numerical approach is found to be efficient, accurate, and numerically stable for the simulation of fluid flows with heat and mass transfer in presence of volumetric radiation in participating medium.

Key words: LBM, RTE, RB convection.

1. Introduction

The lattice Boltzmann method (LBM) has emerged as an efficient method to analyze a vast range of problems in fluid flow and heat transfer (Chen, 1999, Succi, 2001, Wang, 2013, Jiaung, 2001, Mishra, 2005, Chaabane, 2011a, Chaabane, 2011b, Mondal, 2009a, Mishra, 2009, Chaabane, 2011c, Asinari, 2010 and Di Rienzo, 2011), because it uses simple microscopic kinetic models to stimulate complex transport phenomena.

The use of the LBM to formulate and solve different types of heat transfer problems involving volumetric radiation in different geometries has been extended (Mishra, 2014a, Mishra, 2014a, Chen, 1999, Succi, 2001, Wang, 2013, Jiaung, 2001, Mishra, 2005, Chaabane, 2011a, Chaabane, 2011b, Mondal, 2009a, Mishra, 2009, Chaabane, 2011c) but the radiative information was computed using the conventional CFD-RTE solvers.

The present paper deals with the solution of a coupled transient Rayleigh Bénard convection and radiation heat transfer problem in a participating rectangular geometry where the computations of the radiative information and the solution of Navier-Stokes equations are done using the LBM.

2. Dynamic equations

The governing lattice Boltzmann equation is given by (Succi, 2001) for the density and velocity

$$f_k(\vec{r}+\vec{c_k}\Delta t, t+\Delta t) = f_k(\vec{r},t) - \frac{\Delta t}{\tau_v}[f_k(\vec{r},t) - f_k^{eq}(\vec{r},t)] + \Delta t\, F, \quad k = 0,..,b \tag{1}$$

where f_k are the particle distribution function defined for the finite set of the discrete particle velocity vectors $\vec{c_k}$. The collision term Ω_k on the right-hand side of Eq. (1) uses the so called BGK approximation (Wang, 2013, Jiaung, 2001). f_k^{eq} is the local equilibrium distribution function that has an appropriately prescribed functional dependence on the local hydrodynamic properties and τ_v is the relaxation time . F represents the external force term.

For the D2Q9 lattice used in the present work, the relaxation time τ_v is defined as (Chaabane, 2011a, and Chaabane, 2011b):

$$\tau_v = \frac{1}{2} + \frac{3\nu}{c^2 \Delta t} \tag{2}$$

Where c is the lattice speed $c = \sqrt{3RT}$.

The kinetic viscosity ν appearing in Eq. (2) is computed from the Prandtl number $\Pr = \nu / \alpha$ and Rayleigh number $Ra = g\beta_T (T_h - T_c) H^3 / \alpha\nu$. T_h is the hot wall temperature, T_c is the cold wall temperature, α is the thermal diffusivity, g is the acceleration due to gravity, H is the height of the cavity. It is to be noted that viscosity is selected to insure that Mach number is within the limit of incompressible flow (Mishra and Mishra, 2014a, 2014b).

The macroscopic density ρ and the velocity $\vec{u}$ are calculated as follow:

$$\rho(\vec{r},t) = \sum_k f_k(\vec{r},t) \tag{3}$$

$$\vec{u}(\vec{r},t) = \sum_k \vec{c}_k f_k(\vec{r},t) / \rho(\vec{r},t) \tag{4}$$

3. Thermal equations

The governing lattice Boltzmann equation for the thermal field and the volumetric radiation are given by (Chaabane, 2011a, Chaabane, 2011b, Mondal, 2009a, Mishra, 2009):

$$g_k(x+\Delta x, y+\Delta y, t+\Delta t) = (1 - (\frac{\Delta t}{\tau_T}))g_k(x,y,t) + (\frac{\Delta t}{\tau_T})\, g_k^{eq}(x,y,t) - (\frac{\Delta t}{\rho c_p})w_k \nabla.\vec{q_R} \tag{5}$$

where g_k is the particle distribution function denoting the evolution of the internal energy, α is the thermal diffusivity, τ_T is the relaxation time and $\vec{q_R}$ is the radiative heat flux. $\alpha = k / \rho C_p$ is the thermal diffusivity. g_k^{eq} is the equilibrium particle distribution function. Temperature is calculated from the equation of state, $e = RT$

$$\mathrm{T}(\vec{r},t) = \sum_k g_k(\vec{r},t) \tag{6}$$

The divergence of radiative heat flux appearing in Eq. (11) is given by

$$\nabla \overrightarrow{q_R} = \mathrm{k}_a[4\pi(\frac{\sigma \mathrm{T}^4}{4}) - \mathrm{G}] \tag{7}$$

Where k_a, is the absorption coefficient and G is the incident radiant energy. It is convenient to consider a pseudo-transient equation as the starting point of the LBM formalism where the transient RTE is rewritten as

$$\frac{1}{c}\frac{\partial I_i}{\partial t} + s_i.\nabla I_i = \beta(\frac{G}{4\pi} - I_i) \tag{8}$$

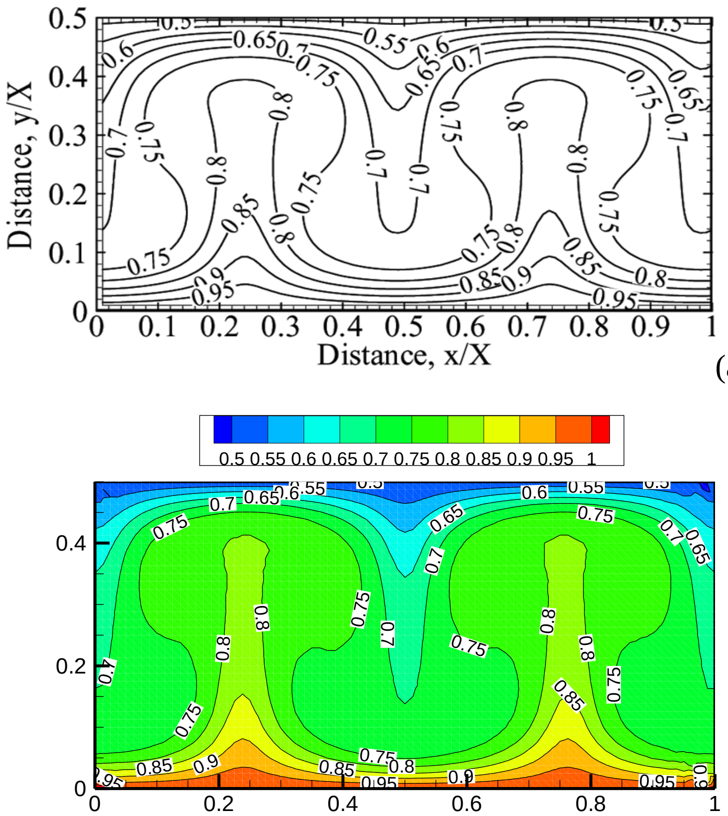
Equation (8) can be rewritten as (Asinari, 2010 and Di Rienzo, 2011):

$$\frac{1}{\Delta t}[\mathrm{I}_i(\mathrm{x}_n + V_i\Delta t, t+\Delta t) - \mathrm{I}_i(\mathrm{x}_n, t)] = \omega_i[\mathrm{I}_i^{eq}(\mathrm{x}_n, t)] - \mathrm{I}_i(\mathrm{x}_n, t)] + \mathrm{O}(\Delta \mathrm{t}) + \mathrm{O}(\mathrm{V}_i\,\Delta \mathrm{t}) \tag{9}$$

I_i^{eq} is the equilibrium distribution function and G is the incident radiation.

4. Results

Our recent approach will be validated against results presented in (Mishra, 2014b) where momentum and energy equations are formulated and solved using the lattice Boltzmann method (LBM) but the volumetric radiative information needed in the energy equation is computed using the Finite Volume Method (FVM). Figure 1 shows isotherms contours for Pr=0.71, Ra=25000, β=1, $\omega = 0$, time step=20000 and RC=250. In all cases a good agreement was achieved.



(a)

(b)

Fig. 1: Isotherms for Pr=0.71, Ra=25000, β=1, $\omega = 0$, time step=20000 and RC=250, (a) reference(Mishra2010b), (b) present work.

4. Conclusions

With the effects of radiation on RB convection, an LBM code was validated for different cluster values with the results available in literature. All results were found to provide accurate results. This non-coupled and non-hybrid numerical approach has the advantage can be extended to other complex engineering heat and flow transfer problems including more sophisticated geometry.

5. References

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