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ADJOINT LATTICE BOLTZMANN METHOD FOR THERMAL FLOW TOPOLOGY OPTIMIZATION

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ABSTRACT

This communication aims to present a topology optimization method for thermo-fluid flow problems. The cost function gradient is computed via an adjoint-state Lattice Boltzmann Method. The LBM Forward problem for fluid flow and heat transfer is written as:

$$\begin{aligned}
\frac{\partial f_i}{\partial t} + \boldsymbol{c_i} \cdot \nabla f_i + \frac{1}{\tau}\left(f_i - f_i^{eq}\right) = 0 \quad \forall x \in X,\ \forall t \in I,\ \forall i \in C \\
\frac{\partial g_i}{\partial t} + \boldsymbol{c_i} \cdot \nabla g_i + \frac{1}{\tau_g}\left(g_i - g_i^{eq}\right) = 0 \quad \forall x \in X,\ \forall t \in I,\ \forall i \in C
\end{aligned}$$

The optimization problem is defined with the design parameter α:

$$\min\, j(\alpha) = \bar{J}(\boldsymbol{f}, \boldsymbol{g}, \alpha) \text{ subject to } Y \leq 0$$

The following inner product is introduced, for conciceness of the derivation that follows:

$$(u, v)_{X\times I\times C} = \sum_i \int_X \int_I u_i(\boldsymbol{x}, t)\ v_i(\boldsymbol{x}, t)\ \mathrm{d}t\ \mathrm{d}\boldsymbol{x} = (\boldsymbol{u}, \boldsymbol{v})_{X\times I}$$

The cost function gradient is defined as:

$$j^{'}(\alpha\,; \delta_\alpha) = \left(\frac{\partial \bar{J}}{\partial \alpha}, \delta_\alpha\right)_X + \left(\frac{\partial \bar{J}}{\partial \boldsymbol{g}}, \frac{\partial \boldsymbol{g}}{\partial \alpha}\delta_\alpha\right)_X + \left(\frac{\partial \bar{J}}{\partial \boldsymbol{f}}, \frac{\partial \boldsymbol{f}}{\partial \alpha}\delta_\alpha\right)_X \tag{1}$$

The cost function gradient is rewritten by introducing adjoint-states $\boldsymbol{g}^* = \{g_i^*\}$ and $\boldsymbol{f}^* = \{f_i^*\}$:

$$\begin{aligned}
j^{'}(\alpha\,; \delta_\alpha) = \left(\frac{\partial \bar{J}}{\partial \alpha}, \delta_\alpha\right)_X + \left(\frac{\partial \boldsymbol{R}^f}{\partial \alpha}\delta_\alpha, \boldsymbol{f}^*\right)_{X\times I} + \left(\frac{\partial \boldsymbol{P}^f}{\partial \alpha}\delta_\alpha, \boldsymbol{f}^*\right)_{\partial X\times I} \\
+ \left(\frac{\partial \boldsymbol{R}^g}{\partial \alpha}\delta_\alpha, \boldsymbol{g}^*\right)_{X\times I} + \left(\frac{\partial \boldsymbol{P}^g}{\partial \alpha}\delta_\alpha, \boldsymbol{g}^*\right)_{\partial X\times I}
\end{aligned} \tag{2}$$

$\boldsymbol{R}^f = \{R_i^f\}$ and $\boldsymbol{R}^g = \{R_i^g\}$ are the LBM residuals. $\boldsymbol{P}^f = \{P_i^f\}$ and $\boldsymbol{P}^g = \{P_i^g\}$ contain the

LBM boundary conditions. Then following the work carried out by Favennec et al. [2016] in a different context , the ALBE for $\boldsymbol{f^*}$ and $\boldsymbol{g^*}$ are obtained by identification between (1) and (2):

$$-\frac{\partial f_i^*}{\partial t} - \boldsymbol{c_i} \cdot \nabla f_i^* + \frac{1}{\tau}\left(f_i^* - f_i^{eq,*}\right) + Q_i = 0 \quad \forall x,\ \forall t,\ \forall i \tag{3}$$

$$-\frac{\partial g_i^*}{\partial t} - \boldsymbol{c_i} \cdot \nabla g_i^* + \frac{1}{\tau_g}\left(g_i^* - g_i^{eq,*}\right) + Q_i^g = 0 \quad \forall x,\ \forall t,\ \forall i \tag{4}$$

The optimization method is validated with a 2D thermal flow in a domain of 100×100 [mm] (Figure 1). The fluid is water and the solid is steel. The flow is laminar and positive heat fluxes are prescribed on top and on bottom of the domain. The boundary conditions are given on Figure 2. The optimization objective is the mean temperature minimization subject to pressure drop constraint ($\Delta P_{\max} = C_{\max}\Delta P_{\text{ini}}, C_{\max} = 20$ for the following results). As we can see on Figure 4, the solid is introduced to bring the fluid near to the hot top and bottom walls.

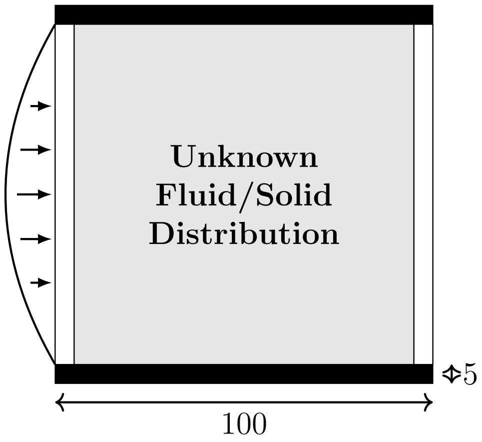

Figure 1: Problem configuration

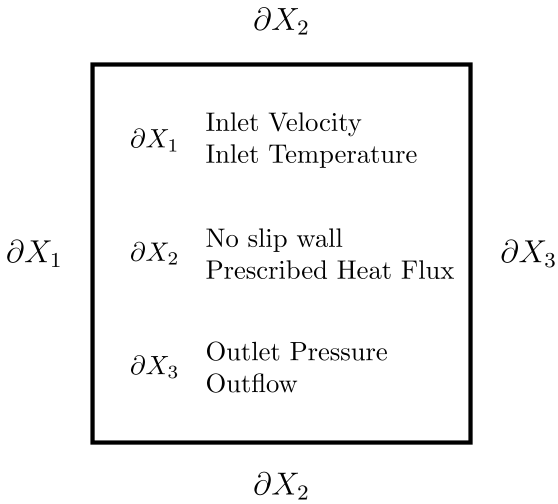

Figure 2: Boundary Conditions

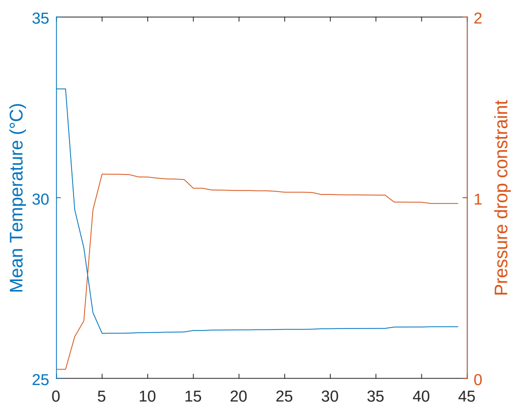

Figure 3: Convergence of the cost function and the constraint

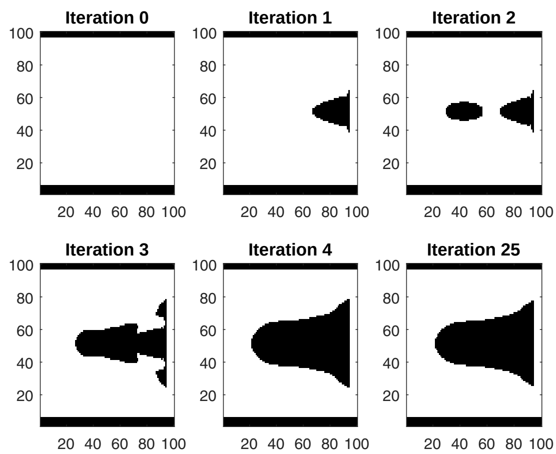

Figure 4: Evolution of the geometry during optimization iterations

REFERENCES

Favennec, Y., Dubot, F., Le Hardy, D., Rousseau, B., and Rousse, D. [2016]. Space-dependent sobolev gradients as a regularization for inverse radiative transfer problems. *Mathematical Problems in Engineering*.