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FINITE ELEMENT ANALYSIS OF HEAT EXCHANGER NETWORK

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ABSTRACT

The Finite Element Method is mathematical techniques for obtaining approximate numerical solutions to the abstract equations of calculus that predict the response of physical systems subjected to external influences. The finite element method is based on the idea of building a complicated object with simple blocks or dividing a complicated object into small and manageable pieces.

Heat exchanger network is the system where many heat exchangers are interconnected in series, parallel, or combined series-parallel through which many fluids are flowing. A typical example of heat exchanger network is shown in Figure-1 and symbolically it is represented in Figure-2. It consists of three heat exchangers and four fluids.

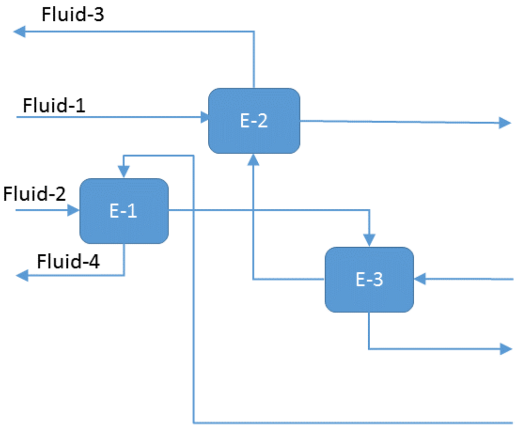

Figure-1 Heat Exchanger Network

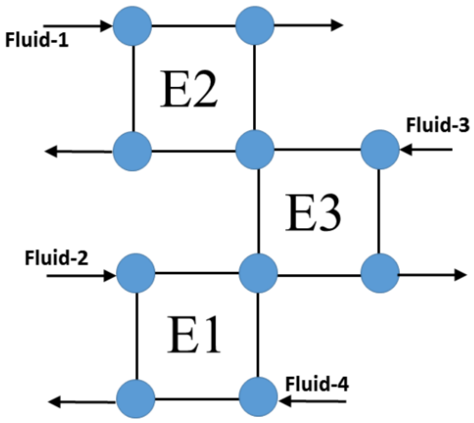

Figure-2 Symbolic Representation

In the present work basic methodology of finite element method was relate with heat exchanger network. This network was considered as finite element model and each exchanger was considered as element. Each stream (hot/cold and in/out) were considered as node points.

Heat exchanger network simulation model is needed to find out the temperature distribution in the heat exchanger network. So far for this purpose trial and error method is being used. In present work, for mathematical formulation of the heat exchanger network, a generalized heat exchanger network simulation model is developed. This model will be applicable for any number of heat exchangers, any number of fluids and for given interconnectivity of heat exchangers in the network. This work is application of pinch technology.

To develop simulation model for heat exchanger network, single heat exchanger simulation model is developed by using energy balance equations and effectiveness-NTU method. Thus generalized model to find out outlet temperature of each fluid in the heat exchanger in the network is developed.

The simulation model is tested for practical heat exchange network and simulation results are compared. It is found that the model results are within the range of ±1% deviation from practical value.

INTRODUCTION

Energy conservation by means of network of heat exchangers is a common feature in process plants. The effective design of these networks depends on an understanding of basic thermodynamic system as well as mechanical limitations involved.

The problem of heat exchanger design is solved in two steps. The first step involves the synthesis of network for minimum cost and second step involves the optimization of the network.

Design procedures are well established for exchangers with simple flow arrangements. Similar procedure does not exist for simulating the exchangers connected in series, parallel or complex flow arrangements.

Mikhailov and Ozisik[1] introduced the procedures followed in finite element analysis to obtain the temperature distribution. They extended the model for the analysis of network of heat exchangers.

In the present paper a generalized heat exchanger network simulation model is introduced by using FEM approach, assuming each heat exchanger as element and four points of streams (hot in/out and cold in/out) are nodes. The new method to find out temperature distribution in the heat exchanger network by formulation of mathematical model for exchanger network simulation is described. The application of this equation in each heat exchanger network analysis is illustrated and discussed.

To analyze and simulate the network each exchanger in the network will be considered individually. To do this energy balance and thermodynamic relations are used. The input parameters include the physical configuration of individual heat exchanger in the network, the inlet fluid properties and the interconnectivity of heat exchangers in the network. The temperatures are calculated for each heat exchanger at average fluid temperature in the heat exchanger.

In many practical cases combination of heat exchangers is used in series, parallel mixed. The heat exchangers are often of non-identical type (shell and tube, double pipe, plate and frame, cross flow etc.) or size and a multiplicity of cases are possible.

In simulation of heat exchanger network, heat balance equations for each heat exchanger will be considered. To generate the heat balance equations two methods are available, one is LMTD approach and other is effectiveness-NTU approach. In the present section, effectiveness-NTU approach is used and the features of both approaches are explained.

To generate the heat balance equations by effectiveness method, one has to know the minimum heat capacity between the heat exchanger fluids, the heat capacity of both fluids and inlet temperatures of the fluids. The effectiveness and minimum heat capacity for different types of heat exchangers can be calculated by thermal and hydraulic design handbook.

LMTD APPROACH VS EFFECTIVENESS-NTU APPROACH

If the inlet fluid conditions and fluid properties along with heat transfer coefficient are known, outlet temperatures of each fluid in the heat exchanger will be find out as explained below.

Consider a parallel flow heat exchanger as shown in Figure-3 having T_1, T_2 and T_3, T_4 are the inlet and outlet temperatures of hot and cold fluid respectively. W_1 and W_2 are the fluid heat capacities of the fluids 1 and 2 respectively. ΔT_i and ΔT_o are the temperature differences at the inlet and outlet of the heat exchanger.

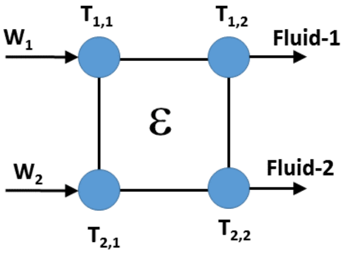

Figure-3 Parallel flow heat exchanger

Now, amount of heat transfer (Q) is given as follows:

$$Q = W_1 (T_1 - T_3) \qquad \ldots(1)$$

$$Q = W_2 (T_4 - T_2) \qquad \ldots(2)$$

by LMTD approach

$$Q = UA\, \Delta T_{LM} \qquad \ldots(3)$$

$$\text{where} \quad \Delta T_{LM} = \frac{\Delta T_i - \Delta T_o}{\ln\left(\dfrac{\Delta T_i}{\Delta T_o}\right)} \qquad \ldots(4)$$

Equation 3 is non-linear equation. There are three simultaneous equations of heat balance in the simulation of the heat exchanger, two are linear and one is non-linear. The solution of this type of simultaneous equations is difficult. Secondly, if iterative method is used for solution of these equations, the method fails as the initial temperature drop for both inlet and outlet are same. Hence it is difficult to use LMTD approach for network simulation.

By Effectiveness-NTU method

$$Q = \varepsilon Q_{max} \qquad \ldots(5)$$

where 'Q_{max}' is the maximum possible heat transfer and 'ε' is the effectiveness of heat exchanger.

Equation 5 is the linear equation. If effectiveness-NTU method is used all three equations (viz. equation 1, 2 and 5) in the linear form. Solution of these equations is easier and Gauss Seidal iterative method can be used for this purpose. Hence in the present work effectiveness-NTU method is used.

SIMULATION OF SINGLE HEAT EXCHANGER

Considering single heat exchanger as shown in Figure-4 having $T_{1,1}$ and $T_{2,1}$ as the inlet temperatures of fluid 1 and 2 respectively. W_{min} is the minimum heat capacity of the fluid in the heat exchanger (i.e. W_h or W_c whichever is minimum). Let Q is the heat duty of the heat exchanger and 'ε' is the effectiveness of the heat exchanger.

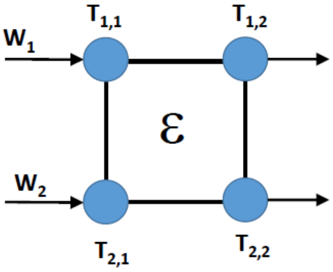

Figure-4 Single Heat Exchanger

Case 1:

Consider fluid 1 is hot fluid and fluid 2 is cold fluid. Then heat balance equations are written as

$$Q = W_1 (T_{1,1} - T_{1,2}) \qquad ...(6)$$

$$Q = W_2 (T_{2,2} - T_{2,1}) \qquad ...(7)$$

$$\varepsilon = \frac{W_1 (T_{1,1} - T_{1,2})}{W_{min}(T_{1,1} - T_{2,1})} \qquad ...(8)$$

$$\varepsilon = \frac{W_2 (T_{2,2} - T_{2,1})}{W_{min}(T_{1,1} - T_{2,1})} \qquad ...(9)$$

Combining equations 6 and 7 it can be written as

$$W_1 (T_{1,1} - T_{1,2}) = W_2 (T_{2,2} - T_{2,1}) \qquad ...(10)$$

Combining equations 8 and 10 it can be written as

$$T_{2,2} = T_{2,1} + \frac{\varepsilon W_{min}}{W_2} (T_{1,1} - T_{2,1}) \qquad ...(11)$$

Combining equations 9 and 10 it can be written as

$$T_{1,2} = T_{1,1} - \frac{\varepsilon W_{min}}{W_1} (T_{1,1} - T_{2,1}) \qquad ...(12)$$

Rearranging the terms in the equations 11 and 12 one can get

$$T_{1,2} = \frac{T_{1,1}(W_1 - \varepsilon W_{min}) + \varepsilon W_{min} T_{2,1}}{W_1} \qquad ...(13)$$

$$T_{2,2} = \frac{T_{2,1}\left(W_2 - \varepsilon W_{\min}\right) + \varepsilon W_{\min} T_{1,1}}{W_2} \qquad \ldots(14)$$

Case 2:
Consider now fluid 1 is cold fluid and fluid 2 is hot fluid. Then the heat balance equations are written as follows

$$Q = W_1 \left(T_{1,2} - T_{1,1}\right) \qquad \ldots(15)$$

$$Q = W_2 \left(T_{2,1} - T_{2,2}\right) \qquad \ldots(16)$$

$$\varepsilon = \frac{W_1 \left(T_{1,2} - T_{1,1}\right)}{W_{\min}\left(T_{2,1} - T_{1,1}\right)} \qquad \ldots(17)$$

$$\varepsilon = \frac{W_2 \left(T_{2,1} - T_{2,2}\right)}{W_{\min}\left(T_{2,1} - T_{1,1}\right)} \qquad \ldots(18)$$

Combining equations 15, 16 and 17 it can be written as

$$T_{2,2} = \frac{T_{2,1}\left(W_2 - \varepsilon W_{\min}\right) + \varepsilon W_{\min} T_{1,1}}{W_2} \qquad \ldots(19)$$

Combining equations 15, 16 and 18 it can be written as

$$T_{1,2} = \frac{T_{1,1}\left(W_1 - \varepsilon W_{\min}\right) + \varepsilon W_{\min} T_{2,1}}{W_1} \qquad \ldots(20)$$

From the above discussion we can write the generalized formula for finding out the outlet temperature in the heat exchanger in terms of heat capacity of fluid, effectiveness and inlet temperatures of fluids as follows:

$$T_{\text{fluid, Outlet}} = \frac{T_{\text{fluid,inlet}}\left(W_{\text{fluid}} - \varepsilon W_{\min}\right) + \varepsilon W_{\min} T_{\text{otherfluid, inlet}}}{W_{\text{fluid}}} \qquad \ldots(21)$$

The illustrative examples of heat exchanger networks, where heat exchangers are arranged in series flow, counter flow and complex flow arrangements will be considered to find a generalized formula for outlet temperature of the fluids at each node in the network.

SERIES FLOW ARRANGEMENT

Consider two heat exchangers with series flow as shown in Figure-5.

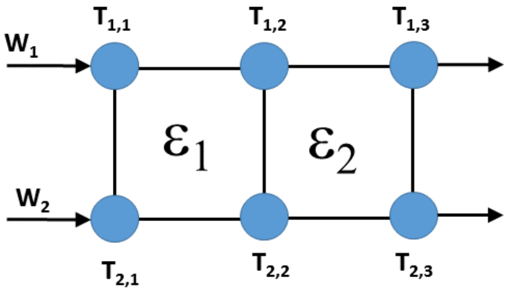

Figure-5 Heat Exchangers with series flow arrangement

Applying the equation 21 to heat exchanger 1 it can be written as

$$T_{1,2} = \frac{T_{1,1}\left(W_1 - \varepsilon_1 W_{\min 1}\right) + \varepsilon_1 W_{\min 1} T_{2,1}}{W_1} \qquad \ldots(22)$$

$$T_{2,2} = \frac{T_{2,1}\left(W_2 - \varepsilon_1 W_{\min 1}\right) + \varepsilon_1 W_{\min 1} T_{1,1}}{W_2} \qquad \ldots(23)$$

In the equations 22 and 23 'ε_1' is the effectiveness of heat exchanger 1 and $W_{\min 1}$ is the minimum heat capacity between the heat exchanger fluids. All the properties are to be taken at average of inlet and outlet temperatures in the heat exchangers.

Similarly outlet temperatures of heat exchanger 2 can be written as follows:

$$T_{1,3} = \frac{T_{1,2}\left(W_1 - \varepsilon_2 W_{\min 2}\right) + \varepsilon_2 W_{\min 2} T_{2,2}}{W_1} \qquad \ldots(24)$$

$$T_{2,3} = \frac{T_{2,2}\left(W_2 - \varepsilon_2 W_{\min 2}\right) + \varepsilon_2 W_{\min 2} T_{1,2}}{W_2} \qquad \ldots(25)$$

Similarly, fluid properties in equations 24 and 25 are to be taken at average fluid temperature in the heat exchanger 2.

From equations 22 to 25 the general form for outlet temperature of each fluid in heat exchanger with series flow arrangement can be written as given in equation 26.

$$T_{\mathrm{fluid,outlet}} = \frac{T_{\mathrm{fluid,inlet}}\left\{W_{\mathrm{fluid}} - \varepsilon_{\mathrm{hx}}\left(W_{\min}\right)_{\mathrm{hx}}\right\} + \varepsilon_{\mathrm{hx}}\left(W_{\min}\right)_{\mathrm{hx}} T_{\mathrm{otherfluid,inlet}}}{W_{\mathrm{fluid}}} \qquad \ldots(26)$$

COUNTER FLOW ARRANGEMENT

Consider two heat exchangers arranged in counter flow as shown in Figure-6

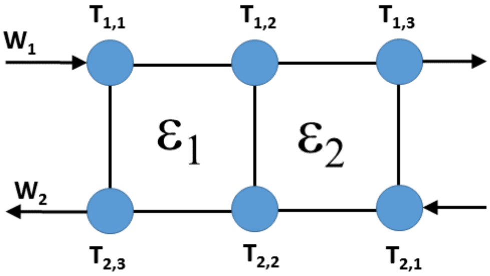

Figure-6 Heat Exchangers in counter flow arrangement

Applying the equation 21 to heat exchanger 1 and 2 then, it can be written as

$$T_{1,2} = \frac{T_{1,1}\left(W_1 - \varepsilon_1 W_{\min 1}\right) + \varepsilon_1 W_{\min} T_{2,2}}{W_1} \qquad \ldots(27)$$

$$T_{2,3}=\frac{T_{2,2}\left(W_2-\varepsilon_1 W_{min1}\right)+\varepsilon_1 W_{min1}T_{1,1}}{W_2} \quad \ldots(28)$$

$$T_{1,3}=\frac{T_{1,2}\left(W_1-\varepsilon_2 W_{min2}\right)+\varepsilon_2 W_{min2}T_{2,1}}{W_1} \quad \ldots(29)$$

$$T_{2,2}=\frac{T_{2,1}\left(W_2-\varepsilon_2 W_{min2}\right)+\varepsilon_2 W_{min2}T_{1,2}}{W_2} \quad \ldots(30)$$

Similarly, fluid properties in equations 21 to 24 are to be taken at average fluid temperature in the heat exchanger.

From equations 27 to 30 the general form for outlet temperature of each fluid in heat exchanger for counter flow arrangement can be written same as given in equation 26.

$$T_{fluid,outlet}=\frac{T_{fluid,inlet}\left\{W_{fluid}-\varepsilon_{hx}\left(W_{min}\right)_{hx}\right\}+\varepsilon_{hx}\left(W_{min}\right)_{hx}T_{otherfluid,inlet}}{W_{fluid}}$$

COMPLEX FLOW ARRANGEMENT

Considering the example of heat exchangers with series and counter fluid flow arrangement in the network as shown in Figure-7.

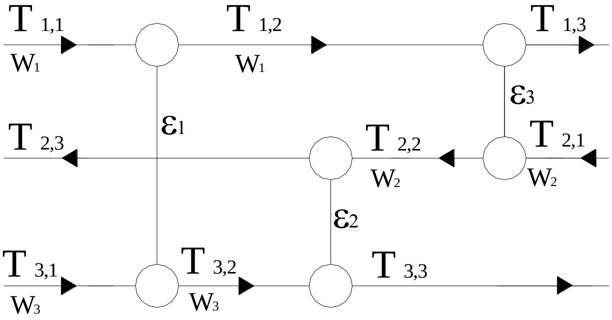

Figure-7 Complex heat exchanger network

Applying equation 21 to individual heat exchangers in this network and writing the outlet temperatures of each heat exchangers as follows:

$$T_{1,2}=\frac{T_{1,1}\left(W_1-\varepsilon_1 W_{min1}\right)+\varepsilon_1 W_{min1}T_{3,1}}{W_1} \quad \ldots(31)$$

$$T_{3,2}=\frac{T_{3,1}\left(W_3-\varepsilon_1 W_{min1}\right)+\varepsilon_1 W_{min1}T_{1,1}}{W_3} \quad \ldots(32)$$

$$T_{2,3}=\frac{T_{2,2}\left(W_2-\varepsilon_2 W_{min2}\right)+\varepsilon_2 W_{min2}T_{3,2}}{W_2} \quad \ldots(33)$$

$$T_{3,3}=\frac{T_{3,2}\left(W_3-\varepsilon_2 W_{min2}\right)+\varepsilon_2 W_{min2}T_{2,2}}{W_3} \quad \ldots(34)$$

$$T_{1,3} = \frac{T_{1,2}\left(W_1 - \varepsilon_3 W_{\min 3}\right) + \varepsilon_3 W_{\min 3} T_{2,1}}{W_1} \qquad \ldots(35)$$

$$T_{2,2} = \frac{T_{2,1}\left(W_2 - \varepsilon_3 W_{\min 3}\right) + \varepsilon_3 W_{\min 3} T_{1,2}}{W_2} \qquad \ldots(36)$$

In all this case also the fluid properties are taken at average fluid temperature in the individual heat exchangers.

From equations 32 to 37 the general form for outlet temperature of each fluid in heat exchanger for complex flow arrangement can be written same as given in equation 26.

$$T_{\text{fluid,outlet}} = \frac{T_{\text{fluid,inlet}}\left\{W_{\text{fluid}} - \varepsilon_{hx}\left(W_{\min}\right)_{hx}\right\} + \varepsilon_{hx}\left(W_{\min}\right)_{hx} T_{\text{otherfluid,inlet}}}{W_{\text{fluid}}}$$

GENERALIZED FORM OF OUTLET TEMPERATURE

From the discussion for series flow, counter flow and complex flow arrangement, the generalized form is obtained by equation 21 is not sufficient to express the outlet temperature of fluid in the heat exchanger when the heat exchanger is in the network.

Hence by considering various flow arrangements, a general form of outlet temperature of heat exchanger in the network can be written as follows:

$$T_{\text{fluid,outlet}} = \frac{T_{\text{fluid,inlet}}\left\{W_{\text{fluid}} - \varepsilon_{hx}\left(W_{\min}\right)_{hx}\right\} + \varepsilon_{hx}\left(W_{\min}\right)_{hx} T_{\text{otherfluid,inlet}}}{W_{\text{fluid}}}$$

For each heat exchanger, two linear simultaneous equations can be written in the form of equation 26 for outlet temperatures of two fluids.

If network consists of ‘n’ heat exchangers, then ‘2n’ number of simultaneous equations will results in. The fluid properties in these equations must be taken at average of inlet and outlet temperature of fluid in the heat exchanger.

Using numerical technique method, this ‘2n’ number of linear simultaneous equations can be solved.

SOLUTION METHOD

As initially the intermediate temperatures in the network are not known, considering average temperature of fluid is same as inlet temperature, take fluid properties at average temperature of each fluid in the heat exchanger. Using fluid properties calculate effectiveness, $W_{\min}$, and W for each fluid for each heat exchanger in the network.

By using generalized equation for outlet temperature, generate a set of linear simultaneous equations. Solve these equations by using Gauss-Seidel iterative method. This gives approximate outlet temperature of each fluid in each heat exchanger in the network.

Now, by using this approximate outlet temperatures in the network find out average temperature of each fluid in each heat exchanger in the network. Now same procedure is repeated as explained above till accuracy is reached.

ILLUSTRATIVE EXAMPLE

Method of solution of heat exchanger network simulation model is explained by considering an illustrative example of heat exchanger network shown in Figure-8. In this exchanger network 2 identical exchangers connected in series and in counter flow. Other input information is given in Table-1.

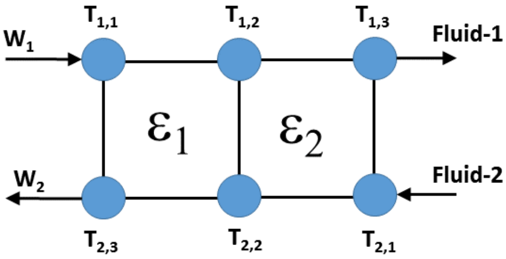

Figure-8 Example

Table-1: Input information for Heat exchangers

		Unit	Fluid-1	Fluid-2
Mass flow rate	*m*	*1000-kg/hr*	293.61	614.17
Specific Heat	*Cp*	*kcal/kg-°C*	0.9985	0.9965
Inlet Temperature	*T*	*°C*	65	32
Overall Heat Transfer coefficient	*U*	*kcal/m2-hr-C*	755	755
Heat Transfer Area	*A*	m^2	403	403

Solution:

$W_1 = m_1Cp_1 = 293169.6$ *kcal/hr°C*

$W_2 = m_2Cp_2 = 612020.4$ *kcal/hr°C*

$W_{min} = 293169.6$ *kcal/hr°C*

$N = UA/W_{min} = 1.038$

$C = Cmin/Cmx = W_1/W_2 = 0.479$

Effectiveness of Shell and Tube Heat Exchanger [5]

$$\varepsilon = 2\left\{1 + C + (1+C^2)^{\frac{1}{2}} \frac{1 + Exp\left[-N(1+C^2)^{\frac{1}{2}}\right]}{1 - Exp\left[-N(1+C^2)^{\frac{1}{2}}\right]}\right\}^{-1}$$

$\varepsilon = 0.5534$

By using equation (27) to (30), intermediate temperature equations are as follows:

$T_{12} = 29.029 + 0.5534\,T_{22}$

$T_{13} = 0.4466\,T_{12} + 17.7088$

$T_{22} = 23.5171 + 0.2651\,T_{12}$

$T_{23} = 0.73491\,T_{22} + 17.2308$

Solution of these equation results:

$T_{12} = 49.2715$, $T_{13} = 39.7135$, $T_{22} = 36.5785$, $T_{23} = 44.1127$

Table-2: Comparison of results

Fluid	Temperature	Mathematical Model	HTRI Output	Deviation %
1	T12	49.2715	49.50	0.5
	T13	39.7135	40.00	0.7
2	T22	36.5785	36.56	0.1
	T23	44.1127	44.00	0.3

CONCLUSIONS

Basic principles of Finite Element Analysis, discretization and applying basic principles to get the elemental solution and then by summation of results complex problem is solved. Same approach is used in the present study. Mathematical model for outlet temperature of heat exchanger based on effectiveness-NTU method is prepared. Network of heat exchangers is divided into number of elements i.e. exchangers. For outlet temperature of each exchanger, developed model is used and generate set of linear equations. Equations are solved using numerical techniques. Final results are compared with commercial software HTRI results and found within ±1% tolerance.

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NOMENCLATURE

Q = Heat Duty
$T_{x,y}$ = Temperature of fluid stream ‘*x*’ at ‘*y*’ point
W_h = Heat capacity of hot fluid.
W_c = Heat capacity of cold fluid.
W_n = Heat capacity of ‘n^{th}’ fluid.
ε = Effectiveness of heat exchanger
E_n = Heat exchanger number ‘n’
C = Heat capacity ratio W_{min}/W_{max}
N = Number of transfer units (NTU)