TURBULENT PRANDTL NUMBER IN THE NEAR-WALL REGION: STUDY OF THE EFFECT OF THE MOLECULAR PRANDTL NUMBER, TRANSPIRATION AND LONGITUDINAL PRESSURE GRADIENT

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Kays [1994] has carried out the analysis of the available by that time experimental data on turbulent Prandtl number Pr_t for the developed flow in a round pipe, a flat channel and for a two-dimensional boundary layer with constant physical properties. The detailed review of numerical studies of the effect of the Reynolds Re and Prandtl Pr numbers on the turbulent heat transfer in the channels of various section at the Re numbers up to 20000 using the DNS and LES techniques has been submitted by Ould-Rouiss [2013]. The results of both experimental and numerical studies have shown that the dependence of the Pr_t number in a logarithmic area has a contradictory character. But generally the turbulent Prandtl number is a function of the molecular Prandtl number Pr, the Reynolds number Re and the distance from the wall y^+: Pr_t (y^+, Pr, Re).

This study presents the numerical solution of the problem of the subsonic stream in the boundary layer on a plate at the values of $Re_x > 5 \times 10^6$. The dependence of the turbulent Prandtl number on the such problem parameters as the molecular Prandtl number Pr, the intensity of a gas transpiration through a permeable wall $j_w = \pm(\rho v)_w/(\rho u)_1$ and the parameter of acceleration (braking) of the main subsonic stream $K = \pm(\mu_1/\rho_1 u_1{}^2) \times (du/dx)$ has been investigated. To close the boundary layer equations system the three-parametrical differential model of turbulence supplemented by the transfer equation for a turbulent heat flux, offered by Lushchik [1988] has been used.

As the gas heat carriers – the air (Pr = 0.71), the mixtures of helium with xenon He-Xe (Pr = 0.21) and helium with argon He-Ar (Pr = 0.41), and as the liquid heat carriers – the mercury Hg (Pr = 0.025), the water H_2O (Pr = 5.9) and the transformer oil (Pr = 88) have been selected. The definition of the task has been accepted as in [Lushchik 2016].

The results of the calculations have shown that for the air the value of Pr_t number is almost constant all over the thickness of the boundary layer and located in the range of the values $Pr_t = 0.85$-0.9, which usually are used in heat transfer calculations. In the liquids (water and transformer oil) with the growth of the molecular Prandtl number the value of Pr_t number in the field of a viscous sublayer ($y^+ \leq 10$) increases. For the gas mixtures with the small values of molecular Prandtl number (Pr = 0.21 and 0.41) the value of Pr_t number also increases in the viscous sublayer, but to a less degree than in the liquids. For the liquid metal (mercury) at Pr = 0.025 the value of Pr_t number significantly changes not only near the wall, but also far from it.

In the range of the injection (suction) intensity values $j_w = 0.. \pm0.005$ on the wall the calculations have shown that at the low suction values its effect on the Pr_t number for the air and gas mixtures is not significant. With the growth of th suction intensity the Pr_t number increases in the logarithmic area and the growth stronger, the lower the molecular Prandtl number. The significant deformation of the speed and temperature profiles is the reason of this growth of the Pr_t number. It has been shown that only for an intensive suction ($j_w = -0.005$) the turbulent Prandtl number considerably differs from the value of $Pr_t \approx 1$. It has been noted that the character of the change of the value of the Pr_t number in the boundary layer significantly depends on the value of the Reynolds number Re_x over the plate length.

The study of the effect of the longitudinal pressure gradient on the turbulent Prandtl number has shown that as the pressure gradient first of all affects on the dynamic characteristics of an the boundary layer (the turbulent friction meaning), at the strong acceleration of a stream it leads to the growth of turbulent Prandtl number in the logarithmic layer.

Since the accuracy of the determination of the Pr_t number values is low, and for the some heat carriers the experimental dependences for the Pr_t number are absent, so for the verification the comparison has been done between the received numerical results and the published experimental data for the parameters entering into the definition of the Pr_t number: the profiles of the temperature, speed, turbulent heat flux and friction. Also the calculation of the friction and heat transfer coefficients (the Nusselt Nu and Stanton St numbers) has been carried out. The received calculation results have been in a good agreement with the experiment.

The study of the effect of the turbulent Prandtl number variability on the characteristics of heat transfer, first of all on the Nusselt number, has shown that the difference of the Nu number defined in the assumption of constancy of the Pr_t number ($Pr_t = 0.85$) from the results received in calculations with using of the heat flux equation increases with the reduction and growth of the molecular Prandtl number concerning the value of $Pr = 0.71$ for the air. The injection, suction, positive and negative pressure gradient also increase this difference which for the studied range of the specified parameters reaches 15%. Thus, if in the experiments the accuracy of determination of the Nu number is higher, then the using of the assumption of the Pr_t number constancy can't be considered to be justified.

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