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SYNTHESIS OF MULTIPERIOD HEAT EXCHANGER NETWORKS WITH A HYBRID META-HEURISTIC APPROACH: SIMULATED ANNEALING AND ROCKET FIREWORKS OPTIMIZATION

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ABSTRACT Optimal Heat Exchanger Network (HEN) designs can lead an industrial plant to significant reduction in operating and capital costs, as well as pollutant emissions and these optimal HENs can be designed with the aid of mathematical programming. An important aspect that can be taken into consideration in the design is the HEN ability of operating under different conditions, and the HEN that meet this feature is known as multiperiod. The synthesis of such HEN, however, requires the solution of a more complex problem. In optimal designs, not only single pieces of equipment must be able to operate under different temperature and flow rates, but also the HEN must perform such task with total annual costs (TAC) as low as possible. In this work, a meta-heuristic approach able to efficiently handle the multiperiod HEN synthesis problem is presented and applied to two case studies. The method could find solutions with better heat loads and stream fraction distributions among the heat exchangers, which led to efficient use of the available heat exchanging area in all periods. The HEN configurations achieved present lower TAC than the solutions reported in the literature, demonstrating the efficiency of the presented meta-heuristic strategy.

INTRODUCTION

Heat exchanger networks (HENs) synthesis has been treated as an essential subject in process engineering literature. The problem consists in arranging the heat exchangers and pairs of streams in such a manner that trade-offs between energy recovery and heat exchanging area lead to minimal costs. Hence, optimal HEN designs are those that, in an industrial plant, can lead to significant reduction in capital and operating costs. As a consequence, pollutant emissions also decrease. Reduction in utilities use may also lead to a higher requirement of total heat exchanging area and number of units, demanding higher capital investments. HEN configurations, thus, must be synthesized considering an optimal trade-off between utilities consumption and capital costs.

Chemical processes might also be multiperiod as a result of changes in operating conditions or products formulation, which imply variations in inlet or outlet temperatures, heat capacities or flow rates. The HEN, thus, must be robust in order to be able to handle multiperiod operation. In case that the HEN is designed for only one period of operation, operating costs might increase or make the process impractical in case of operation variations. Hence, a multiperiod HEN consists of a set of heat exchanging devices subject to uncertainties which are postulated in a finite group of periods.

However, even for plants with fixed operation conditions, the HEN synthesis problem is not straightforward to solve. Early methods relied much on the designers' experience, such as the Pinch technology [e.g., Linnhoff and Hindmarsh, 1983]. More recently, authors aimed for more automated approaches, and mathematical programming became an important tool for HEN synthesis. Mixed-integer nonlinear programming (MINLP) formulations can be proposed to achieve good HEN configurations when properly optimized. These kind of formulations need superstructures, such as the hyperstructure of Floudas and Ciric [1989], which is a rather complete but complex to solve model, and the stage-wise superstructure (SWS) of Yee and Grossmann [1990], which is simpler. Nonetheless, even with simplifying assumptions, HEN mathematical models are laborious to solve. The complexity arises from the large number of possible combinations of heat exchange matches, as well as non-linearities and non-convexities.

Furthermore, the necessity of HEN able to operate under conditions that differ depending on periods of the year led researchers to develop approaches taking such consideration into account. Such feature adds complexity to a problem already difficult to solve, and represents a challenge to the HEN synthesis academic community. The work of Floudas and Grossmann [1987] pioneered the mathematical programming based multiperiod HEN synthesis topic with a sequential approach. Later, Aaltola [2002] presented a formulation based on the SWS. In his model, capital cost was calculated with the average of the required areas in each period. Considering the larger area value to each piece of equipment among the values achieved for all periods would be more realistic, since in that case, operating conditions would be always fulfilled. The average area assumption, however, avoided the numerical difficulties of a non-differentiable *max* function, which should find the largest required area for heat exchanger operation among all periods. That consideration under-estimated TAC, and was not assumed in the work of Verheyen and Zhang [2006], which was able to present more realistic costs. Jiang and Chang [2013] presented a model able to overcome equipment over-sizing by rearranging stream pairs passing through each heat exchanger. The model was re-worked by Miranda et al. [2016a], who were able to present better results. Miranda et al. [2016b] presented a sequential model which improves the formulation of Floudas and Grossmann [1987].

In order to obtain optimal HEN using the aforementioned mathematical programming formulations, the solution approach must be reliable and able to avoid local-minima trapping efficiently. To the day, no method was able to guarantee global optimality to the most acknowledged MINLP models [e.g., Floudas and Ciric, 1989; Yee and Grossmann, 1990] and variants for single-period HEN synthesis in medium and large-scale cases. Better results to a series of *benchmark* problems are constantly being reported in the literature, tough, demonstrating that the solution approaches, as well as processing technologies, are evolving. Solution methodology can be deterministic or stochastic. The former is based on advanced mathematical concepts, such as gradients and hessian, to perform their optima search. The latter relies on randomized disturbances that are controllably applied to problem variables, and on the iterative re-evaluation of the objective function with heuristic acceptance rules to improve the solutions. These approaches are also called meta-heuristics, and are an important trend in the literature. Numerous works have presented hybrid meta-heuristic methods for single-period HEN synthesis. Some featured works use methods such as combinatorial and continuous Genetic Algorithms (GA) [e.g., Lewin, 1998], GA with Pinch technology [e.g., Ravagnani et al., 2005], monogenetic algorithm [e.g., Luo et al., 2009], Simulated Annealing (SA) and GA with Particle Swarm Optimization (PSO) and parallel processing [e.g., Pavão et al., 2016a; Pavão et al., 2017], and SA with Rocket

Fireworks Optimization (RFO) [e.g., Pavão et al., 2016b]. Stochastic methodologies have been applied to multiperiod HEN synthesis as well, especially with GA and SA based methodologies [e.g., Ma et al., 2008; Yi et al., 2013].

This work proposes to adapt the SA-RFO hybrid methodology for single-period HEN synthesis from Pavão et al. [2016b] in order to make it able to handle multiperiod HEN synthesis. The original methodology has been able to provide solutions with costs lower than the current literature. The scheme is based on the two-level HEN synthesis concept. SA is responsible for binary variables optimization, while RFO, which is a novel method presented in that work, which consists in the application of Continuous Simulated Annealing (CSA) prior to PSO, handles the continuous domain. The reader is referred to that work for results comparison as well as for more detailed HEN synthesis literature review.

FORMULATION

In the present study, multiperiod HEN is modeled after the acknowledged stage-wise superstructure (SWS) [e.g., Yee and Grossmann, 1990]. The model comprises all possible matches in each stage, but only one heat exchanger per branch of split stream. The original SWS model considers that mixers inlet temperatures are equal (isothermal mixing assumption), but it is not considered in this work. Hence, an extra constraint of heat balance for mixers is present. The SWS is represented in Figure 1. Another constraint which forbids heat exchanger areas smaller than 1.0 m^2 [e.g., Miranda et al., 2016a] was included. Such condition avoids results that are impractical from the design point of view. The proposal of Verheyen and Zhang [2006] of using the largest areas for each period was incorporated to the objective function as well.

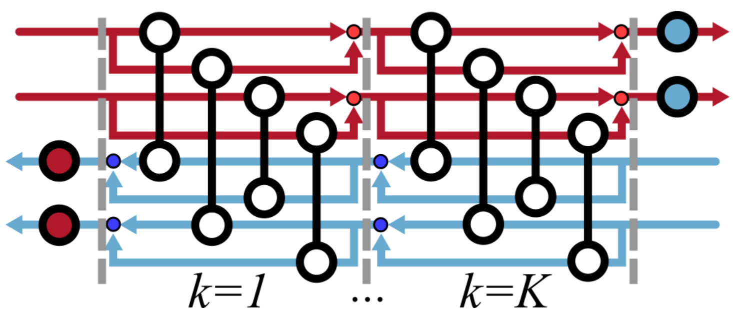

Figure 1. Stage-wise superstructure

The objective function to be minimized is the Total Annualized Cost (TAC). This TAC is the sum of annualized capital and HEN operating costs and is formulated as follows:

min TAC

$$\begin{aligned} TAC \quad &= \sum_p \sum_i Ccu \cdot \frac{t_p}{ttotal} \cdot Qcu_{p,i} + \sum_p \sum_j Chu \cdot \frac{t_p}{ttotal} \cdot Qhu_{p,j} + \\ &AF \cdot \left(\sum_i \sum_j \sum_k z_{i,j,k} \cdot (B + C \cdot \max_p (A_{p,i,j,k})^{\beta}) + \sum_i zcu_i \cdot (B + C \cdot \max_p (Acu_{p,i})^{\beta}) + \right. \\ &\left. \sum_j zhu_j \cdot (B + C \cdot \max_p (Ahu_{p,j})^{\beta}) \right), p \in N_P, i \in N_H, j \in N_C, k \in N_S \end{aligned} \tag{1}$$

where *TAC* is the total annual cost of the HEN, *Ccu* and *Chu* are utilities costs ("*cu*" and "*hu*" suffixes stand for cold and hot utilities in all variables); *Qcu* and *Qhu* are the energy exchange requirements in utilities for each period and stream; *z*, *zcu* and *zhu* are binary variables which indicate the existence of a heat exchange device (the *p* index is not necessary in the binary variables because if a device exists for a certain pair of streams in a period, it will exist with the same pair of streams in the other periods); *B* are the fixed capital costs for each unit; *C* is the capital costs factor, β is the capital costs exponent; *A*, *Acu* and *Ahu* are areas of each piece of equipment, calculated as follows.

$$A_{p,i,j,k} = \frac{z_{i,j,k} Q_{p,i,j,k}}{U_{p,i,j,k} LMTD_{p,i,j,k}}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \tag{2}$$

$$Acu_{p,i} = \frac{zcu_i Qcu_{p,i}}{Ucu_{p,i} LMTDcu_{p,i}}, p \in N_P, i \in N_H \tag{3}$$

$$Ahu_{p,j} = \frac{zhu_j Qhu_{p,j}}{Uhu_{p,j} LMTDhu_{p,j}}, p \in N_P, j \in N_C \tag{4}$$

Please note that, in order to consider multiperiod operation, some variables must be adapted. Total utilities use (*Qhu* and *Qcu*) are now bi-dimensional matrixes that consider all periods and are weighted according to the duration of each operation time span. The variables *t* and *ttotal* are, respectively, the hours of operation of a period *p* and the total hours of operation in a year. Areas are calculated for each period as well, and a *p* index is also added to *A*, *Acu* and *Ahu* variables. Please, notice also that a "*max*" function in *p* index is required to find largest area among all periods for each match. Thus, in some periods, heat exchangers might be over-sized. In that manner, multiperiod HEN that uses most of their heat exchange area in all periods are usually able to present lower costs.

Mixers energy balances, which are an important improvement from the original SWS [e.g., Yee and Grossmann, 1990], are a feature of the model used in this work that is worth stressing. In the original proposal, outlet temperatures of heat exchangers in two or more branches of a same split stream in the same stage are equal. This assumption avoids a nonlinear energy balance for mixers. However, some minima are likely missed with such consideration. That is the reason in this work energy balance is performed for every mixer, according to Eqs (5) and (6).

$$Tmixh_{p,i,k} = Tmixh_{p,i,k-1} - \sum_j \frac{z_{i,j,k} \cdot Q_{p,i,j,k}}{CPh_{p,i}}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \tag{5}$$

$$Tmixc_{p,j,k} = Tmixc_{p,j,k+1} + \sum_i \frac{z_{i,j,k} \cdot Q_{p,i,j,k}}{CPc_{p,j}}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \tag{6}$$

where, *Tmixh* and *Tmixc* are mixers outlet temperatures in each stage of the SWS, *Q* are exchanger heat loads and *CPh* and *CPc* are streams total heat capacities. Please notice the *p* subscripts in those variables, which means that the balances must be performed in all periods.

Energy balances to obtain outlet temperatures are performed according to Eqs (7) and (8).

$$Thout_{p,i,k} = Tmixh_{p,i,k-1} - \frac{z_{i,j,k} \cdot Q_{p,i,j,k}}{Fh_{p,i,j,k} CPh_{p,i}}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \quad (7)$$

$$Tcout_{p,j,k} = Tmixc_{p,j,k+1} + \frac{z_{i,j,k} \cdot Q_{p,i,j,k}}{Fc_{p,i,j,k} CPc_{p,j}}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \quad (8)$$

It is worth noting that stream fractions of each branch (*Fh* and *Fc*) are variables that can be changed in each period of operation. Specific calculations for outlet heat exchanger temperatures, logarithmic mean temperature differences, heaters and coolers energy balances, areas and feasibility constraints are analogous to those presented in the formulation section of Pavão et al. [2016b]. Main differences regard the consideration of different periods of operation in the variables.

SOLUTION APPROACH

In the present paper, the solution approach is a revisited version of SA-RFO [e.g., Pavão et al., 2016b]. The method was able to provide solutions with lower TAC than many other works from current literature for single-period HEN, encouraging the development of adaptations to make the method able to handle other more elaborated formulations, such as multiperiod HEN. The methodology uses the concept of two-level HEN synthesis. Under such approach, the "outer" level meta-heuristic must perform disturbances in the HEN structural level, i.e., the match binary variable that contains information on whether a heat exchanger of the superstructure exists or not ($z_{i,j,k}$). At each topology disturbance, an "inner" level continuous optimization method must be applied in order to search for heat loads ($Q_{p,i,j,k}$) and stream fractions ($Fh_{p,i,j,k}$ and $Fc_{p,i,j,k}$) that are able to lead to an optimal trade-off between heat exchanging area and utility costs.

Simulated annealing (SA) is an acknowledged combinatorial optimization approach developed by Kirkpatrick et al. [1983]. It relies on "moves" performed to a single solution, which are accepted or not according to a simple set of rules. If the solution is better than the current one, it is accepted. Otherwise, its acceptance depends on the Metropolis acceptance rule [e.g., Metropolis et al., 1953]. With such rule, depending on the method's parameters tuning, a solution with higher costs than the current one might be accepted. This simple strategy aids in avoiding premature stagnation in local minima. Besides SA-RFO, SA has served as basis for numerous other HEN synthesis approaches [e.g., Athier et al., 1997; Dolan et al., 1989; Pavão et al., 2017; Peng and Cui, 2015]. In the present work, the SA level performs only a simple move of adding one single heat exchanger to the HEN structure, i.e., "flipping" a zero in the $z_{i,j,k}$ matrix to one. The continuous optimization approach, which is presented further, is then applied to that topology in the "inner" level, and returns its optimal costs to SA, in the "outer" level, which takes the decision regarding the new topology's acceptance. A heat exchanger is only removed from the topology if the inner level returns a configuration where one of the heat loads to a match is zero in all periods.

Rocket Fireworks Optimization (RFO) is a continuous optimization method that consists of two steps. In the first one, a variation of SA for continuous spaces (Continuous Simulated Annealing, CSA) is applied. A match in a period is randomly selected, and an also random quantity within a pre-determined range is added or removed to its heat load or stream fraction, according to Eqs. (9) to (11).

$$Q_{p,i,j,k} \leftarrow Q_{p,i,j,k} + Q_{move}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \quad (9)$$

$$Fh_{p,i,j,k} \leftarrow Fh_{p,i,j,k} + Fh_{move}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \tag{10}$$

$$Fc_{p,i,j,k} \leftarrow Fc_{p,i,j,k} + Fc_{move}, p \in N_P, i \in N_H, j \in N_C, k \in N_S \tag{11}$$

In the second step, random solutions are generated in the feasible region, mimicking an "explosion". The solution achieved by CSA is likely already in a promising region. It is maintained and incorporated to the set of new random solutions that serve as initial swarm for PSO. In PSO, as proposed by Kennedy and Eberhart [1995], initial particles are set with random velocities and are guided and updated by the best solutions found by the swarm and by themselves. Incorporating a good initial solution to the swarm improves much the algorithm performance. Particles are able to find better solutions in the region of that first, refining the results. That initial solution in the present algorithm, as already mentioned, is the CSA solution. It is very likely much better than the other random solutions, taking place as the swarm leader and guiding other solutions towards its already promising region. Moreover, particles may also find good regions on their path towards the leader. Figure 2 presents an illustration of the solutions behavior over a minimum region during (a) CSA, (b) random particles generation and (c) PSO.

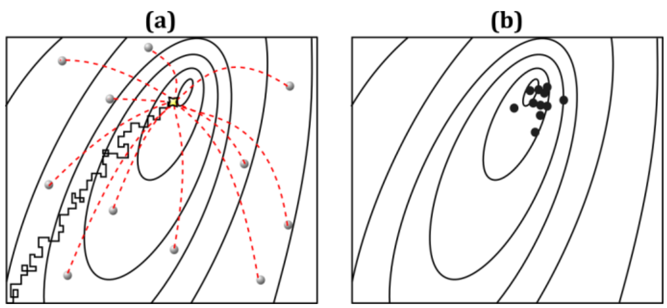

Figure 2. Solutions behavior during Rocket Fireworks Optimization. (a) Continuous simulated annealing and random swarm generation (c) Particle swarm optimization

For more specific information regarding the algorithms choice, parameters tuning and explanations, equations and rules for solutions updating in the SA-RFO method, as well as constraints handling strategies, the reader is referred to Pavão et al. [2016b].

CASE STUDIES

The developed method was implemented in C++ using Microsoft Visual Studio 2015, which is a free integrated development environment. It was applied to two case studies from the literature in order to test its efficiency. Total annual costs comparison is presented in each example. The tests were carried out on a computer with a 3.5 GHz Intel i5 processor and 8 GB of RAM.

Case study 1. This is an example with two hot and two cold streams, and was proposed by Jiang and Chang [2013]. Data is presented in Table 1. It is worth noting that, differently from most examples in the literature, not only streams inlet temperatures and total heat capacities are uncertain, but outlet temperatures and heat transfer coefficients change depending on the operation period as well.

Table 1. Case study 1 stream data

Stream	Tin (K)			Tout (K)			CP (kW/K)			h (kW/(m^{2}K))		
Period	1	2	3	1	2	3	1	2	3	1	2	3
H1	650	630	645	370	380	350	10.0	10.2	10.0	1.00	1.03	1.01
H2	590	570	600	370	340	350	20.0	20.5	20.3	1.00	1.04	1.04
C1	410	390	420	640	630	660	15.0	15.0	14.3	1.00	1.02	1.05
C2	350	340	320	500	520	540	13.0	13.5	13.0	1.00	1.05	1.03

Area costs = $4333 \cdot A^{0.6}$; Annualizing factor = 0.1/yr
Chu (680-680 K) = 150.163 \$/(kWyr); Ccu (300-320 K) = 53.064 \$/(kWyr);
h_{HU} = 5 kW/(m^{2}K); h_{CU} = 1 kW/(m^{2}K)

The model used by Jiang and Chang [2013] to synthesize multiperiod HEN comprised switching stream pairs that passed through a heat exchanger in order to reduce over-sizing issues. Miranda et al. [2016a] improved that model and corrected some solution inconsistencies, being able to find better results with more efficient use of the available heat exchanging area. The basis formulation used in the present work does not comprise streams switching, but the improved SA-RFO method was able to achieve a configuration with lower *TAC* and six units. Solution approaches used by previous authors relied in a separate HEN synthesis to each period, while the presented in this work considers all periods simultaneously, which makes the method able to provide HEN with more usage of the available heat exchanging area in all operating conditions. The optimal HEN was achieved in 84 min and is presented in Figure 3. A comparison to the literature is presented in Table 2, which presents, besides *TAC*, the number of units, total area, capital costs (*CC*) and operating costs (*OC*). More detailed design data regarding this HEN configuration, such as inlet and outlet temperatures for each piece of equipment, can be found in Table A.1 of the Appendix section. It is worth noting that, by means of an efficient heat loads and stream fractions distribution, a solution with greater area, but smaller utilities requirement was achieved. Also, it is worth stressing out that the present solution has the same number of units as in previous works solutions, but one less split (solutions by Jiang and Chang [2013] and Miranda et al. [2016a] have splits in H2 and C2) and no stream pairing switching. That means, apart from being less expensive, the present solution is also simpler to implement in practice.

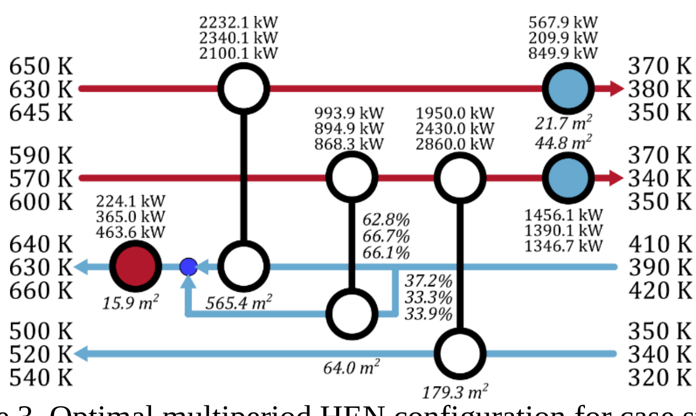

Figure 3. Optimal multiperiod HEN configuration for case study 1

Table 2. Results comparison for case study 1

	Devices	*Area* (m^2)	*CC* (\$/yr)	*OC* (\$/yr)	*TAC* (\$/yr)
Jiang and Chang [2013]	6	521	33,627	171,656	205,283
Miranda et al. [2016a]	6	498	33,202	171,656	204,858
This study	6	891	43,685	155,646	199,331

Case study 2. This example was presented by Aaltola [2002], who modified the six hot and one cold stream problem originally proposed by Floudas and Grossmann [1987]. Problem data is presented in Table 3. In this case, uneven periods are considered. The plant operates 75% of time under nominal conditions, while the remaining 25% operation time is divided equally among conditions of periods 1-3. The total operation time per year is 8760 h, as assumed by Aaltola [2002].

Table 3. Case study 2 stream data

Stream	*Tin* (°C)				*Tout* (°C)	*CP* (kW/°C)			
Period	Nominal	1	2	3	All	Nominal	1	2	3
H1	630	640	620	620	460	9.00	9.90	9.90	8.10
H2	550	560	540	540	480	6.50	7.15	7.15	5.85
H3	530	540	520	520	480	3.00	3.30	3.30	2.70
H4	470	480	460	460	400	36.00	39.60	39.60	32.40
H5	450	460	440	440	310	7.00	7.70	7.70	6.30
H6	410	420	400	400	350	72.00	79.20	79.20	64.80
C1	310	300	300	300	650	27.00	29.70	29.70	24.30

Area costs = $4333 \cdot A^{0.6}$;
Furnace costs = $191.94 \cdot qhu^{0.7}$, qhu [=] kW, no further sizing data is necessary for heater related *CC*;
$Chu = 204.73 \cdot 10^{-4}$ \$/(kWh); Ccu (300-330°C) = $60.58 \cdot 10^{-4}$ \$/(kWh);
$U_{1,1} = 0.6$; $U_{2,1} = 0.4$; $U_{3,1} = 0.3$; $U_{4,1} = 0.4$; $U_{6,1} = 0.3$; $Ucu_1 = 0.1$; $Ucu_5 = 0.3$; $Ucu_6 = 0.4$
$U_{i,j}$ and Ucu_i [=] kW/(m^{2}K)

The solution of Aaltola [2002] had inconsistencies regarding its capital costs. Moreover, the author approximates area costs using the average required area of all periods to avoid numerical issues of the non-differentiable *max* function. His solution was revised and recalculated using the maximum areas of each heat exchanging unit and the correct LMTD equation instead of the approximation [e.g., Chen, 1987] used by the author. Despite the slightly higher *TAC* found in the revision, Aaltola's [2002] solution is still an interesting multiperiod HEN configuration. The present method achieved the very same HEN topology, but was able to achieve a more economic configuration by more efficiently distributing heat loads and stream fractions over that topology. The present configuration is slightly more costly regarding utilities, which is compensate by smaller areas, leading to lower total annual costs. The processing time was of 75 min. The HEN configuration is depicted in Figure 4. Solutions comparison is presented in Table 4. For other design data, please see Table A.2, in the Appendix.

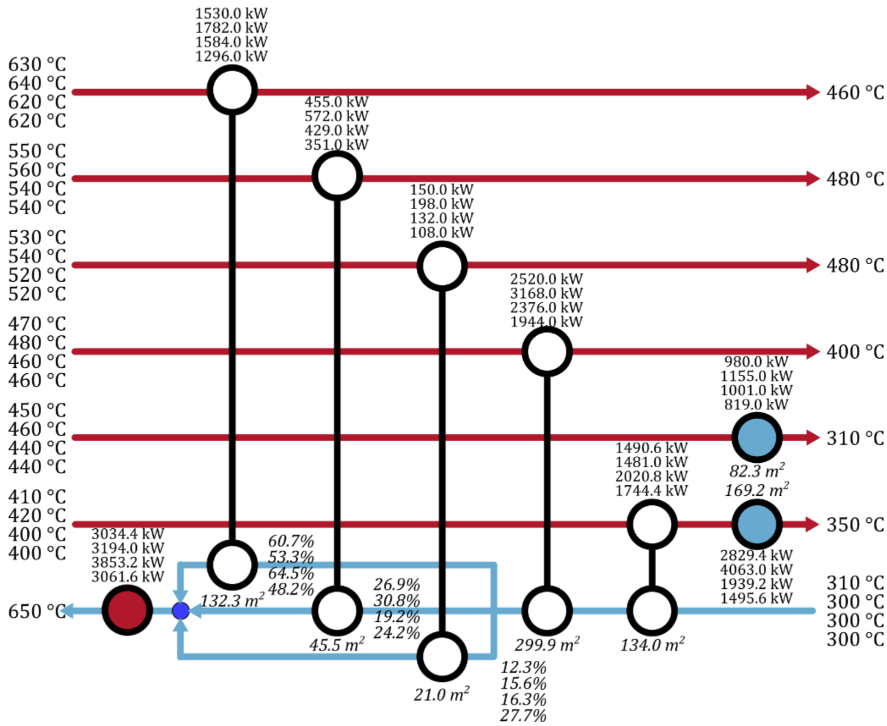

Figure 4. Optimal multiperiod HEN configuration for case study 2

Table 2. Results comparison for case study 2

	Devices	*Area* (m^2)	*CC* ($/yr)	*OC* ($/yr)	*TAC* ($/yr)
Aaltola [2002]*	8	913	260,696	754,250	1,014,946
This study	8	884	254,898	757,162	1,012,061

*Revised solution

CONCLUSIONS

A meta-heuristic method for multiperiod HEN synthesis was presented. The approach was based on the SA-RFO method [e.g., Pavão et al., 2016b], which is a two-level scheme for single-period HEN synthesis. It uses SA to propose new HEN structures in the binary level and RFO to find optimal heat loads and stream fractions in the continuous level. The multiperiod HEN synthesis mathematical programming model employed was based on the SWS of Yee and Grossmann [1990]. Moreover, simplifications such as the isothermal mixing and LMTD approximations were not assumed. The solution approach was able to achieve better configurations than those presented in the literature in both cases studied. The HEN synthesized in the first case study was compared to solutions obtained with a formulation, which comprised streams switching. Remarkably, the present algorithm found a configuration with lower costs. In the second case study, a previous solution from the literature was firstly revised. The present method was then applied to the problem, achieving the same HEN structure with different heat loads and stream fractions. Such more efficient distributions demonstrate especially the potentialities of the continuous optimization approach (RFO). Furthermore, it is worth noting that the present strategy attempts to find one HEN configuration for all periods simultaneously, instead of synthesizing HEN for each period separately and then applying an algorithm to find the final multiperiod HEN. With such features and the results achieved, it can be concluded that the presented method has proven efficient. Promising topologies and optimal heat loads and stream fractions among matches were found, leading networks to have more useful heat exchanging area and lower costs throughout all periods of operation.

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NOMENCLATURE

Variables/Parameters:

A	[m^2]	Heat exchanger area
Acu	[m^2]	Cooler area
AF	[-]	Annualizing factor
Ahu	[m^2]	Heater Area
B	[$/yr]	Fixed heat exchanger cost
C	[$/(m$^{2\beta}$yr)]	Heat exchanger cost factor
Ccu	[$/(kWyr) or $/(kWh)]	Cold utility cost
Chu	[$/(kWyr) or $/(kWh)]	Hot utility cost
CP	[kW/K]	General process stream total heat capacity
CPc	[kW/K]	Cold stream total heat capacity
CPh	[kW/K]	Hot stream total heat capacity
Fc	[-]	Cold stream fraction
Fh	[-]	Hot stream fraction
h	[kW/(m^{2}K)]	General process stream heat transfer coefficient
h_{CU}	[kW/(m^{2}K)]	Cold utility heat transfer coefficient
h_{HU}	[kW/(m^{2}K)]	Hot utility heat transfer coefficient
LMTD	[K or °C]	Logarithmic mean temperature difference
LMTDcu	[K or °C]	Cooler logarithmic mean temperature difference
LMTDhu	[K or °C]	Heater logarithmic mean temperature difference
max	[-]	Function that returns the maximum value along a given set
Q	[kW]	Heat exchanger heat load
Qcu	[kW]	Total cold utilities heat load
qhu	[kW]	Furnace heat load
Qhu	[kW]	Total hot utilities heat load

TAC	[$/yr]	Total annual cost
Tcout	[K or °C]	Heat exchanger cold stream outlet temperature
Thout	[K or °C]	Heat exchanger hot stream outlet temperature
Tin	[K or °C]	General process stream inlet temperature
Tmixc	[K or °C]	Cold stream mixer outlet temperature
Tmixh	[K or °C]	Hot stream mixer outlet temperature
Tout	[K or °C]	General process stream outlet temperature
t	[h/yr]	Plant operation time under period *p* conditions per year
ttotal	[h/yr]	Plant operation time per year
U	[kW/(m^{2}K)]	Global heat transfer coefficient
Ucu	[kW/(m^{2}K)]	Cooler global heat transfer coefficient
Uhu	[kW/(m^{2}K)]	Heater global heat transfer coefficient
z	[-]	Heat exchanger existence binary variable
zcu	[-]	Cooler existence binary variable
zhu	[-]	Heater existence binary variable
β	[-]	Capital cost exponent

Indexes:

i	[-]	Hot stream
j	[-]	Cold stream
k	[-]	Stage
p	[-]	Period

Sets:

N_C	[-]	Cold streams
N_H	[-]	Hot streams
N_P	[-]	Periods
N_S	[-]	Stages

APPENDIX

Table A.1. Case study 1 optimal HEN design data

Match (p,i,j,k)		(p,1,1,2)	(p,2,1,2)	(p,2,2,3)	(p,1,CU)	(p,2,CU)	(p,HU,1)
A_{max}	m²	565.4	64.0	179.3	21.7	44.8	15.9
Period 1 (p = 1)							
A/A_{max}	%	100.0	100.0	34.6	60.2	69.1	36.0
$Q_{p,i,j,k}$	kW	2232.1	993.9	1950.0	567.9	1456.1	224.1
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.627	0.372	1.0			1.0
$Tmixh_{p,i,k-1}$	K	650.0	590.0	540.3	426.8	442.8	680.0
$Tmixh_{p,i,k}$	K	426.8	540.3	442.8	370.0	370.0	680.0
$Thout_{p,i,j,k}$	K	426.8	540.3	442.8	370.0	370.0	680.0
$Tmixc_{p,j,k+1}$	K	410.0	410.0	350.0	300.0	300.0	625.1
$Tmixc_{p,j,k}$	K	625.1	625.1	500.0	320.0	320.0	640.0
$Tcout_{p,i,j,k}$	K	647.1	587.9	500.0	320.0	320.0	640.0
Period 2 (p = 2)							
A/A_{max}	%	100.0	98.6	100.0	23.8	100.0	44.2
$Q_{p,i,j,k}$	kW	2340.1	894.9	2430.0	209.9	1390.1	365.0
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.667	0.333	1.0			1.0
$Tmixh_{p,i,k-1}$	K	630.0	570.0	526.3	400.6	407.8	680.0
$Tmixh_{p,i,k}$	K	400.6	526.3	407.8	380.0	340.0	680.0
$Thout_{p,i,j,k}$	K	400.6	526.3	407.8	380.0	340.0	680.0
$Tmixc_{p,j,k+1}$	K	390.0	390.0	340.0	300.0	300.0	605.7
$Tmixc_{p,j,k}$	K	605.7	605.7	520.0	320.0	320.0	630.0
$Tcout_{p,i,j,k}$	K	624.0	569.0	520.0	320.0	320.0	630.0
Period 3 (p = 3)							
A/A_{max}	%	100.0	93.8	67.1	100.0	83.4	100.0
$Q_{p,i,j,k}$	kW	2100.1	868.3	2860.0	849.9	1346.7	463.6
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.661	0.339	1.0			1.0
$Tmixh_{p,i,k-1}$	K	645.0	600.0	557.2	435.0	416.3	680.0
$Tmixh_{p,i,k}$	K	435.0	557.2	416.3	350.0	350.0	680.0
$Thout_{p,i,j,k}$	K	435.0	557.2	416.3	350.0	350.0	680.0
$Tmixc_{p,j,k+1}$	K	420.0	420.0	320.0	300.0	300.0	627.6
$Tmixc_{p,j,k}$	K	627.6	627.6	540.0	320.0	320.0	660.0
$Tcout_{p,i,j,k}$	K	642.2	599.0	540.0	320.0	320.0	660.0

Table A.2. Case study 2 optimal HEN design data

Match (p,i,j,k)		(p,1,1,2)	(p,2,1,2)	(p,3,1,2)	(p,4,1,3)	(p,6,1,4)	(p,5,CU)	(p,6,CU)	(p,HU)
Amax	m²	132.2	45.5	21.0	299.9	134.0	82.3	169.2	-
Nominal Conditions (p = 0)									
A/A_{max}	%	100.0	100.0	100.0	100.0	61.4	89.7	76.7	-
$Q_{p,i,j,k}$	kW	1530.0	455.0	150.0	2520.0	1490.6	980.0	2829.4	3034.4
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.608	0.269	0.122	1.0	1.0			1.0
$Tmixh_{p,i,k-1}$	°C	630.0	550.0	530.0	470.0	410.0	450.0	389.3	537.6
$Tmixh_{p,i,k}$	°C	460.0	480.0	480.0	400.0	389.3	310.0	350.0	650.0
$Thout_{p,i,j,k}$	°C	460.0	480.0	480.0	400.0	389.3	310.0	350.0	650.0
$Tmixc_{p,j,k+1}$	°C	458.5	458.5	458.5	365.2	310.0	300.0	300.0	-
$Tmixc_{p,j,k}$	°C	537.6	537.6	537.6	458.5	365.2	330.0	330.0	-
$Tcout_{p,i,j,k}$	°C	551.8	521.1	503.8	458.5	365.2	330.0	330.0	-
Period 1 (p = 1)									
A/A_{max}	%	100.0	100.0	100.0	75.2	43.4	100.0	100.0	-
$Q_{p,i,j,k}$	kW	1782.0	572.0	198.0	3168.0	1480.9	1155.0	4063.0	3194.0
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.535	0.308	0.156	1.0	1.0			1.0
$Tmixh_{p,i,k-1}$	°C	640.0	560.0	540.0	480.0	420.0	460.0	401.3	542.4
$Tmixh_{p,i,k}$	°C	460.0	480.0	480.0	400.0	401.3	310.0	350.0	650.0
$Thout_{p,i,j,k}$	°C	460.0	480.0	480.0	400.0	401.3	310.0	350.0	650.0
$Tmixc_{p,j,k+1}$	°C	456.5	456.5	456.5	349.9	300.0	300.0	300.0	-
$Tmixc_{p,j,k}$	°C	542.4	542.4	542.4	456.5	349.9	330.0	330.0	-
$Tcout_{p,i,j,k}$	°C	568.6	519.0	499.1	456.5	349.9	330.0	330.0	-
Period 2 (p = 2)									
A/A_{max}	%	51.9	100.0	55.0	97.3	100.0	97.2	60.7	-
$Q_{p,i,j,k}$	kW	1584.0	429.0	132.0	2376.0	2020.7	1001.0	1939.2	3853.2
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.645	0.192	0.163	1.0	1.0			1.0
$Tmixh_{p,i,k-1}$	°C	620.0	540.0	520.0	460.0	400.0	440.0	374.5	520.3
$Tmixh_{p,i,k}$	°C	460.0	480.0	480.0	400.0	374.5	310.0	350.0	650.0
$Thout_{p,i,j,k}$	°C	460.0	480.0	480.0	400.0	374.5	310.0	350.0	650.0
$Tmixc_{p,j,k+1}$	°C	448.0	448.0	448.0	368.0	300.0	300.0	300.0	-
$Tmixc_{p,j,k}$	°C	520.3	520.3	520.3	448.0	368.0	330.0	330.0	-
$Tcout_{p,i,j,k}$	°C	530.7	523.2	475.3	448.0	368.0	330.0	330.0	-
Period 3 (p = 3)									
A/A_{max}	%	64.5	68.1	43.9	100.0	92.0	79.5	47.6	-
$Q_{p,i,j,k}$	kW	1296.0	351.0	108.0	1944.0	1744.4	819.0	1495.6	3061.6
$Fh_{p,i,j,k}$		1.0	1.0	1.0	1.0	1.0	1.0	1.0	
$Fc_{p,i,j,k}$		0.481	0.242	0.277	1.0	1.0			1.0
$Tmixh_{p,i,k-1}$	°C	620.0	540.0	520.0	460.0	400.0	440.0	373.1	524.0
$Tmixh_{p,i,k}$	°C	460.0	480.0	480.0	400.0	373.1	310.0	350.0	650.0
$Thout_{p,i,j,k}$	°C	460.0	480.0	480.0	400.0	373.1	310.0	350.0	650.0
$Tmixc_{p,j,k+1}$	°C	451.8	451.8	451.8	371.8	300.0	300.0	300.0	-
$Tmixc_{p,j,k}$	°C	524.0	524.0	524.0	451.8	371.8	330.0	330.0	-
$Tcout_{p,i,j,k}$	°C	562.5	511.6	467.8	451.8	371,8	330.0	330.0	-

REFERENCES

Aaltola, J., 2002. *Simultaneous synthesis of flexible heat exchanger network*. Appl. Therm. Eng. Vol. *22*, No. 8, pp. 907–918. doi:10.1016/S1359-4311(02)00008-X

Athier, G., Floquet, P., Pibouleau, L., Domenech, S., 1997. *Process optimization by simulated annealing and NLP procedures. Application to heat exchanger network synthesis*. Comput. Chem. Eng. Vol. *21*, pp. S475–S480. doi:10.1016/S0098-1354(97)87547-3

Chen, J.J.J., 1987. *Comments on improvements on a replacement for the logarithmic mean*. Chem. Eng. Sci. Vol. *42*, No. 10, pp. 2488–2489. doi:10.1016/0009-2509(87)80128-8

Dolan, W.B., Cummings, P.T., LeVan, M.D., 1989. *Process optimization via simulated annealing: Application to network design*. AIChE J. Vol. *35*, No. 5, pp. 725–736. doi:10.1002/aic.690350504

Floudas, C.A., Ciric, A.R., 1989. *Strategies for overcoming uncertainties in heat exchanger network synthesis*. Comput. Chem. Eng. Vol. *13*, No. 10, pp. 1133–1152. doi:10.1016/0098-1354(89)87017-6

Floudas, C.A., Grossmann, I.E., 1987. *Automatic generation of multiperiod heat exchanger network configuration*. Comput. Chem. Eng. Vol. *11*, No. 2, pp. 123–142.

Jiang, D., Chang, C.T., 2013. *A new approach to generate flexible multiperiod heat exchanger network designs with timesharing mechanisms*. Ind. Eng. Chem. Res. Vol. *52*, No. 10, pp. 3794–3804. doi:10.1021/ie301075v

Kennedy, J., Eberhart, R., 1995. *Particle swarm optimization*, in: Proceedings of ICNN'95 - International Conference on Neural Networks. IEEE, pp. 1942–1948. doi:10.1109/ICNN.1995.488968

Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., 1983. *Optimization by simulated annealing.* Science Vol. *220*, No. 4598, pp. 671–80. doi:10.1126/science.220.4598.671

Lewin, D.R., 1998. *A generalized method for HEN synthesis using stochastic optimization - II. The synthesis of cost-optimal networks*. Comput. Chem. Eng. Vol. *22*, No. 10, pp. 1387–1405. doi:10.1016/S0098-1354(98)00221-X

Linnhoff, B., Hindmarsh, E., 1983. *The pinch design method for heat exchanger networks*. Chem. Eng. Sci. Vol. *38*, No. 5, pp. 745–763. doi:10.1016/0009-2509(83)80185-7

Luo, X., Wen, Q.-Y., Fieg, G., 2009. *A hybrid genetic algorithm for synthesis of heat exchanger networks*. Comput. Chem. Eng. Vol. *33*, No. 6, pp. 1169–1181. doi:10.1016/j.compchemeng.2008.12.003

Ma, X., Yao, P., Luo, X., Roetzel, W., 2008. *Synthesis of multi-stream heat exchanger network for multi-period operation with genetic/simulated annealing algorithms*. Appl. Therm. Eng. Vol. *28*, No. 8–9, pp. 809–823. doi:10.1016/j.applthermaleng.2007.07.015

Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H., Teller, E., 1953. *Equation of State Calculations by Fast Computing Machines*. J. Chem. Phys. Vol. *21*, No. 6, pp. 1087. doi:10.1063/1.1699114

Miranda, C.B., Costa, C.B.B., Caballero, J.A., Ravagnani, M.A.S.S., 2016a. *Heat Exchanger Network Optimization for Multiple Period Operations*. Ind. Eng. Chem. Res. Vol. *55*, No. 39, pp. 10301–10315. doi:10.1021/acs.iecr.6b01117

Miranda, C.B., Costa, C.B.B., Caballero, J.A., Ravagnani, M.A.S.S., 2016b. *Optimal synthesis of multiperiod heat exchanger networks: A sequential approach*. Appl. Therm. Eng. doi:10.1016/j.applthermaleng.2016.10.003

Pavão, L.V., Costa, C.B.B., Ravagnani, M.A.S.S., 2017. *Heat Exchanger Network Synthesis without stream splits using parallelized and simplified simulated Annealing and Particle Swarm Optimization*. Chem. Eng. Sci. Vol. *158*, pp. 96–107. doi:10.1016/j.ces.2016.09.030

Pavão, L.V., Costa, C.B.B., Ravagnani, M.A.S.S., 2016a. *Automated heat exchanger network synthesis by using hybrid natural algorithms and parallel processing*. Comput. Chem. Eng. Vol. *94*, pp. 370–386. doi:10.1016/j.compchemeng.2016.08.009

Pavão, L.V., Costa, C.B.B., Ravagnani, M.A.S.S., Jiménez, L., 2016b. *Large-scale heat exchanger networks synthesis using simulated annealing and the novel rocket fireworks optimization*. AIChE J. doi:10.1002/aic.15524

Peng, F., Cui, G., 2015. *Efficient simultaneous synthesis for heat exchanger network with simulated annealing algorithm*. Appl. Therm. Eng. Vol. *78*, pp. 136–149. doi:10.1016/j.applthermaleng.2014.12.031

Ravagnani, M.A.S.S., Silva, A.P., Arroyo, P.A., Constantino, A.A., 2005. *Heat exchanger network synthesis and optimisation using genetic algorithm*. Appl. Therm. Eng. Vol. *25*, No. 7, pp. 1003–1017. doi:10.1016/j.applthermaleng.2004.06.024

Verheyen, W., Zhang, N., 2006. *Design of flexible heat exchanger network for multi-period operation*. Chem. Eng. Sci. Vol. *61*, No. 23, pp. 7754–7765. doi:10.1016/j.ces.2006.09.028

Yee, T.F., Grossmann, I.E., 1990. *Simultaneous optimization models for heat integration—II. Heat exchanger network synthesis*. Comput. Chem. Eng. Vol. *14,* No. 10, pp. 1165–1184. doi:10.1016/0098-1354(90)85010-8

Yi, D., Han, Z., Wang, K., Yao, P., 2013. *Strategy for synthesis of flexible heat exchanger networks embedded with system reliability analysis*. Chinese J. Chem. Eng. Vol. *21*, No. 7, pp. 742–753. doi:10.1016/S1004-9541(13)60541-8