Localized Double Adaptation Refinement vs Localized Meshless Methods
for Convective Heat Transfer

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It is well known that adaptive FEM techniques can produce highly accurate and computationally efficient solutions. Peraire et al. [1] showed that h-adaption could accurately capture shocks in compressible flow; later efforts by others have demonstrated similar success using h-, as well as p-adaptation techniques in solving heat, mass and momentum transport problems. However, few have used hp-adaptive finite element algorithms to solve fluid flow problems [2]. This method provides for a measurement of the error in the discretization, and adjusts the spatial accuracy to minimize the error or bring it within a predefined amount [3]. The conservative, hp-adaptive technique, along with P-G upwinding, enables very accurate simulation with reduced computational storage requirements.

A double adaptive technique combines h-adaptation (element size varies while the order of the shape function remains constant) and p-adaptation (element size is constant while the order of the shape function is increased to meet the desired accuracy requirement). This adaptive combination has been shown to exhibit exceptional computational accuracy with exponential convergence rates [4]. Detailed descriptions regarding adaptation error estimator and adaptation strategy are provided in Wang and Pepper [2].

A sequence of refinement steps is employed. Three consecutive hp-adaptive meshes are constructed for solving the system equations in order to reach a preset target error: initial coarse mesh, the intermediate h-adaptive mesh, and the final hp-adaptive mesh obtained by applying p-adaptive enrichments on the intermediate mesh. The p-adaptation is carried on when the solution is pre-asymptotic.

Mesh reduction and mesh-free methods are now being used to solve the Navier-Stokes equations. These techniques include the dual reciprocity boundary element method [5], mesh-free method of fundamental solutions [6], and mesh-free local Petrov-Galerkin methods [7]. Kansas’ method [8], which is a domain-type meshless method, was developed by directly collocating radial basis functions (RBFs), especially multiquadric approximations (MQ). The use of MQ was first developed by Hardy [9] as an interpolation method for modeling the earth’s gravitational field. The meshless method, based on radial basis functions (RBFs), is especially effective when using Kansa’s approach [8] with MQ.

The meshless methods mentioned above fall under the category of global meshless methods. Global RBFs meshless methods have some well-known drawbacks, including poor conditioning of the ensuing algebraic set of equations and the occurrence of large matrices resulting from the discretization of the PDEs. While these drawbacks can be addressed, to some extent, by domain decomposition and appropriate preconditioning [10,11], these matrices are still extremely sensitive to the choice of the shape parameters in RBFs. Moreover, care must be taken in the evaluation of the spatial derivatives of the PDEs.

Localized RBF meshless methods [12] can avoid many of these issues. Localized meshless methods have been applied to solve complex problems including convection-diffusion with phase-change, Darcy flow, turbulent flow and heat transfer. Other applications to the incompressible Navier-Stokes equations include work by Divo and Kassab [13] and Djidjeli et al. [14] to compute incompressible viscous flows, while Chen et al. [15] proposed an upwind approach to solve the compressible Navier-Stokes equations. Waters and Pepper [16] examined the features of localized meshless methods versus global methods for calculating convective heat transfer.

The main attractive feature of the localized RBF method is that since the coefficient matrix is the same size as the local sub-domain, which is usually relatively small, only small matrices need to be inverted for all points on the domain. The concern of an ill-conditioned matrix, as well as the computational burden of having to evaluate several large matrices at every step of an iteration or time-marching scheme is mitigated at a cost of repeatedly solving these small linear systems. Further advantages include there being no need to precondition the domain size to ensure a non-singular matrix and minimal memory demands as no global collocation matrix needs to be allocated (only small arrays are stored, one for each of the data centers).
et al. [17] demonstrated the advantages of the localized approach for the case of the diffusion-reaction equation in three-dimensions.

In this paper, we compare these two methods for solving the incompressible Navier-Stokes equations, with convective heat transfer, and examine two classic benchmark problems: (1) natural convection within an enclosure, and (2) convective flow over a backward-facing step. Other applications of the localized meshless approach will also be discussed.

References