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Localized Double Adaptation Refinement vs Localized Meshless Methods for Convective Heat Transfer

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It is well known that adaptive FEM techniques can produce highly accurate and computationally efficient solutions. Peraire et al. [1] showed that h-adaption could accurately capture shocks in compressible flow; later efforts by others have demonstrated similar success using h-, as well as p-adaptation techniques in solving heat, mass and momentum transport problems. However, few have used hp-adaptive finite element algorithms to solve fluid flow problems [2]. This method provides for a measurement of the error in the discretization, and adjusts the spatial accuracy to minimize the error or bring it within a predefined amount [3]. The conservative, hp-adaptive technique, along with P-G upwinding, enables very accurate simulation with reduced computational storage requirements.

A double adaptive technique combines h-adaptation (element size varies while the order of the shape function remains constant) and *p*-adaptation (element size is constant while the order of the shape function is increased to meet the desired accuracy requirement). This adaptive combination has been shown to exhibit exceptional computational accuracy with exponential convergence rates [4]. Detailed descriptions regarding adaptation error estimator and adaptation strategy are provided in Wang and Pepper [2].

A sequence of refinement steps is employed. Three consecutive hp-adaptive meshes are constructed for solving the system equations in order to reach a preset target error: initial coarse mesh, the intermediate h-adaptive mesh, and the final hp-adaptive mesh obtained by applying p-adaptive enrichments on the intermediate mesh. The p-adaptation is carried on when the solution is pre-asymptotic.

Meswh reduction and mesh-free methods are now being used to solve the Navier-Stokes equations. These techniques include the dual reciprocity boundary element method [5], mesh-free method of fundamental solutions [6], and mesh-free local Petrov-Galerkin methods [7]. Kansas' method [8], which is a domain-type meshless method, was developed by directly collocating radial basis functions (RBFs), especially multiquadric approximations (MQ). The use of MQ was first developed by Hardy [9] as an interpolation method for modeling the earth's gravitational field. The meshless method, based on radial basis functions (RBFs), is especially effective when using Kansa's approach [8] with MQ.

The meshless methods mentioned above fall under the category of global meshless methods. Global RBFs meshless methods have some well-known drawbacks, including poor conditioning of the ensuing algebraic set of equations and the occurrence of large matrices resulting from the discretization of the PDEs. While these drawbacks can be addressed, to some extent, by domain decomposition and appropriate pre-conditioning [10,11], these matrices are still extremely sensitive to the choice of the shape parameters in RBFs. Moreover, care must be taken in the evaluation of the spatial derivatives of the PDEs.

Localized RBF meshless methods [12] can avoid many of these issues. Localized meshless methods have been applied to solve complex problems including convection-diffusion with phase-change, Darcy flow, turbulent flow and heat transfer. Other applications to the incompressible Navier-Stokes equations include work by Divo and Kassab [13] and Djidjeli et al. [14] to compute incompressible viscous flows, while Chen et al. [15] proposed an upwind approach to solve the compressible Navier-Stokes equations. Waters and Pepper [16] examined the features of localized meshless methods versus global methods for calculating convective heat transfer.

The main attractive feature of the localized RBF method is that since the coefficient matrix is the same size as the local sub-domain, which is usually relatively small, only small matrices need to be inverted for all points on the domain. The concern of an ill-conditioned matrix, as well as the computational burden of having to evaluate several large matrices at every step of an iteration or time-marching scheme is mitigated at a cost of repeatedly solving these small linear systems. Further advantages include there being no need to precondition the domain size to ensure a non-singular matrix and minimal memory demands as no global collocation matrix needs to be allocated (only small arrays are stored, one for each of the data centers). Islam et al. [17] demonstrated the advantages of the localized approach for the case of the diffusion-reaction equation in three-dimensions.

In this paper, we compare these two methods for solving the incompressible Navier-Stokes equations, with convective heat transfer, and examine two classic benchmark problems: (1) natural convection within an enclosure, and (2) convective flow over a backward-facing step. Other applications of the localized meshless approach will also be discussed.

References

[1]. Peraire, J., Vahdati, M, Morgan, K. and Zienkiewicz, O. C., "Adaptive remeshing for compressible flows", *J. Comp. Phys.* vol., 72, no. 2, pp.449-466, 1987.

[2]. Wang, X. and Pepper, D. W., "hp-adaptive FEM simulations of viscous flow including convective heat transfer", *Num. Heat Transf.*, vol. 51, 6, pp. 491-513, 2007.

[3]. Zienkiewicz O. C., Morgan, K., Satya Sai, B.V.K., Codina, R., Vasquez, M., "A general algorithm for compressible and incompressible flow, Part II, Tests on the explicit form," *Int. J. Num. Methods Fluids,* vol. 20, pp. 887– 913, 1995.

[4]. Guo, B. and Babuska, I., "The h-p Version of the Finite Element Method", Parts 1 and 2, *Comp. Mechanics*, vol. 1, pp. 21-21 and pp. 203-220, 1986.

[5]. Sarler, B. and Kuhn, G., Primitive variables dual reciprocity boundary element method solution of incompressible navier-stokes equations. *Engineering Analysis with Boundary Elements*, 23, pp. 443–455, 1999.

[6] Sarler, B., Towards a mesh-free computation of transport phenomena. *Engineering Analysis with Boundary Elements*, 26, pp. 731–738, 2002.

[7]. Lin, H. and Atluri, S. N., Meshless local petrov galerkin method (mlpg) for convection- diffusion problems. *Computer Modelling in Engineering and Sciences*, 1, pp. 45–60, 2001.

[8]. Kansa, E. J., Multiquadrics-a scattered data approximation scheme with applications to computational fluid dynamics i-surface approximations and partial derivative estimates. *Comp.Math. Appl.*, 19, pp. 127–145, 1990.

[9]. Hardy, R. L., Multiquadric equations of topography and other irregular surfaces. *J. Geophys. Res.*, 76, pp. 1905–1915, 1971.

[10]. Ling, L. and Kansa, E. J., A least-squares preconditioner for radial basis functions collocation methods. *Advances in Computational Mathematics*, 23, pp. 31–54, 2005.

[11]. Mai-Duy, N. and Tran-Cong, T., Mesh-free radial basis function network methods with domain decomposition for approximation of functions and numerical solution of poisson's equation. *Eng. Anal. Boundary Elem.*, 26, pp. 133–156, 2002.

[12]. Sarler, B. and Vertnik, R., Meshfree explicit local radial basis function collocation method for diffusion problems. *Comput Math Appl*, 51, pp. 1269-1282, 2006.

[13]. Divo, E. and Kassab, A. J., An efficient localized rbf meshless method for fluid flow and conjugate hear transfer. *ASME J Heat Trans*, 129, pp. 124-136, 2007.

[14]. Djidjeli, K., Chinchapatnam, P. P., and Nair, P. B., Meshless rbf collocation for steady incompressible viscous flows. *AIAA paper 2006-3525, AIAA 36th Fluid Dynamics Conference and Exhibit, San Francisco, CA*, June 2006.

[15]. Chen, H. Q., Shu, C., Ding, H., and Wang, T. G., An upwind local rbf-dq method for simulation of inviscid compressible flows. *Comput, Methods Appl. Mech. Eng.*, 194, pp. 2001-2017, 2005.

[16]. Waters, J. and Pepper, D. W., Global versus Localized RBF Meshless Methods for Solving Incompressible Fluid Flow with Heat Transfer, *Num. Heat Transfer*, vol. 68, 3, pp. 185-203, 2015.

[17]. Islam, S., Guangming, Y., and Sarler, B., Assessment of global and local meshless methods based on collocation with radial basis functions for parabolic partial differential equations in three dimensions. *Engineering Analysis with Boundary Elements*, 36(11), pp. 1640-1648, 2012.