

NUMERICAL MODELLING OF THERMO-VIBRATIONAL INSTABILITIES IN SUPERCRITICAL FLUIDS

Deewakar Sharma*[§], Arnaud Erriguible**, Sakir Amiroudine*

* Université Bordeaux, I2M, UMR CNRS 5295, 16 Av. Pey-Berland, 33607 Pessac, France

** Bordeaux INP, I2M, UMR CNRS 5295, 16 Av. Pey-Berland, 33607 Pessac, France

[§]Correspondence author. Email: deewakar.sharma@u-bordeaux.fr

INTRODUCTION

Supercritical fluids may be regarded as intermediate between liquid and gas exhibiting high liquid like density and low gas like viscosity. These fluids exist beyond the liquid-vapor critical point where there the boundary between liquid and gas does not exist. The demand for supercritical fluids has escalated over the past several decades in diverse industrial applications, efficient refrigeration systems, drilling technologies and in high performance rocket propellants [Shen and Zhang, 2013]. This is attributed to the emergence of some atypical thermo-physical properties as one approaches the critical point such as the divergence of the constant-pressure specific heat capacity and compressibility [Shen and Zhang, 2013]. When these fluids are acted upon by longitudinal vibrations, the coupling between thermal boundary layer and longitudinal accelerations lead to onset of instability of thermal boundary layer which is largely governed by the distance from critical point. In experimental study aboard sounding rocket, emergence of fingers depicting destabilization of thermal boundary was observed wherein super-critical CO₂ was subjected to vibrations under weightlessness. Numerical studies pertaining to supercritical fluids subjected to vibrations were initially performed by [Amiroudine and Beysens, 2008] who observed the fingering structure in the thermal boundary layers with different proximities to the critical point. Similar results were also reported when supercritical hydrogen was subjected to longitudinal vibrations in the numerical studies of [Gandikota *et al.*, 2013]. They evaluated the effect of vibrations using three different combinations of boundary conditions thus providing insight into the mechanisms of corner, parametric and Rayleigh vibrational instability.

However, the numerical studies so far have assumed linear equation of state thus limiting the quench or heating conditions. Further, results obtained using linear state equation may be argued owing to peculiar behaviour of thermal physical properties on approaching the critical point. In this study a numerical model is proposed wherein no explicit equation of state is required for evaluation of density. The numerical results are first validated with the experimental results followed by detailed analysis for different thermal conditions (quench rates and proximities to the critical point) and vibrational parameters (frequency and amplitude). A stability plot is presented for Rayleigh-vibrational and parametric instabilities.

PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

In the current study, a 2-D square cavity of side $h = 7\text{mm}$ filled with supercritical hydrogen is subjected to longitudinal vibrations $A\sin(\omega t)$ as represented in Fig. 1. The problem is mathematically modelled using Navier-Stokes equations comprising of conservation of mass, momentum and energy which can be described as following (see [Amiroudine *et al.*, 2014] for more details),

$$\left\{ \begin{array}{l} \rho \frac{D\mathbf{V}}{Dt} = -\nabla[P^0 - \delta t(\frac{1}{\chi_T} + \frac{\beta_P^2 T}{\rho C_v \chi_T^2})\nabla \cdot \mathbf{V} - \delta t \frac{\beta_P}{\rho C_v \chi_T} \nabla \cdot \boldsymbol{\phi}] + \rho A \omega^2 \sin \omega t \mathbf{i} + \nabla \cdot [\mu(\nabla \mathbf{V} + \nabla^t \mathbf{V} - \frac{2}{3} \nabla \cdot \mathbf{V} \bar{\mathbf{I}})] \\ \rho C_v \frac{DT}{Dt} = \nabla \cdot (k \nabla T) - \frac{\beta_P T}{\chi_T} \nabla \cdot \mathbf{V} \\ \rho = \rho^0 e^{-\delta t \nabla \cdot \mathbf{V}} \\ \text{Properties : } \beta_P, \chi_T, C_v, \mu \text{ and } k \end{array} \right.$$

The density is evaluated directly from the conservation of mass and other thermophysical properties are evaluated using Renormalization Group theory. Here P^0 and T^0 are the equilibrium pressure and temperature at a defined time instant. The set of equation The thermo-physical properties are calculated directly from NIST database while the density is evaluated directly from the continuity equation. These equations are solved numerically by a home-made code (Thetis [Amiroudine *et al.*, 2014]).

RESULTS AND DISCUSSIONS

The frequency of vibrations was varied from 5-35 Hz while the distance from critical point varies from 2000mK to 5mK. The walls were subjected to various quench rates (as a percentage of proximity from critical point). However, in order to study parametric instabilities, horizontal walls were considered adiabatic.

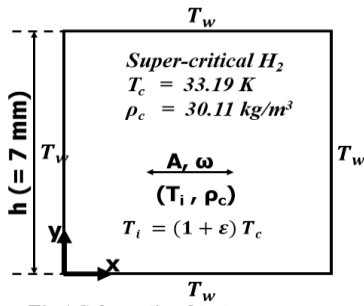


Fig.1 Schematic of test case.

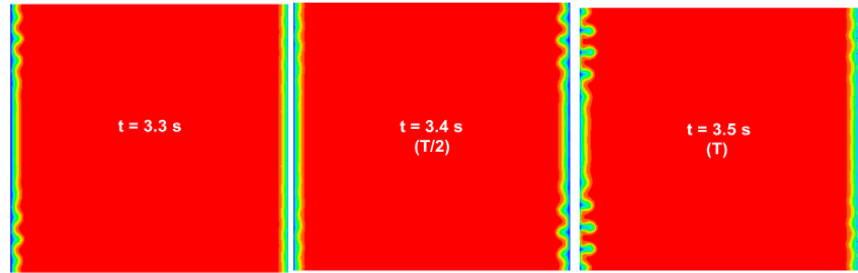


Fig. 2: Time evolution of parametric instabilities ($T-T_c = 500$ mK, $A=20$ mm $f=5$ Hz)

Fig. 2 shows an example of our numerical results for parametric instability for a longitudinal frequency of 5Hz. The time mentioned shows that the instabilities are observed once on each side during each cycle are thus the harmonic in nature. The studies were performed for a period of 50 s and it was observed that depending on the amplitude of vibrations, the instabilities may sustain till the end or die out after some time. A stability curve is thus drawn categorizing three regions, stable, unstable and metastable region for different frequencies.

A similar analysis is carried out for Rayleigh-vibrational instabilities (vibration direction parallel to thermal waves), it was observed that with an increase in the distance from the critical point, the number of fingers decreases while their amplitude grows faster. The latter is attributed to the fact that diffusion time increases significantly as we move closer to the critical point thereby causing a delay in the growth of instability in the thermal boundary layer. In addition, the effect of different quench conditions is also analysed for the above-mentioned cases before drawing a similar stability curve.

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