Proceedings of CHT-17

ICHMT International Symposium on Advances in Computational Heat Transfer

May 28-June 1, 2017, Napoli, Italy

CHT17-172

THERMOCAPILLARITY AND VISCOUS DISSIPATION EFFECTS ON MHD THIN FILM FLOW OF A CASSON FLUID IN A NON-DARCY POROUS MEDIUM WITH VARIABLE THERMAL CONDUCTIVITY

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ABSTRACT: The investigation of hydromagnetic flow of a viscous, electrically conducting, Casson thin fluid film past an unsteady stretching sheet with variable thermal conductivity, under the influence of thermocapillarity and viscous dissipation, for Forchheimer extended Darcy model, is carried out. Similarity transformation is used to translate the governing partial differential equations into ordinary differential equations. Shooting technique in conjunction with Runge-Kutta 4th order method is employed to solve the transformed equations. Detailed computations for fluid velocity inside the thin film and fluid thin film temperature as well as local skin friction co-efficient and local Nusselt number, are carried out for a range of values of pertinent flow parameters to analyze the physics of the film flow. It is observed that thermocapillarity enhance the velocity of fluid near the free surface. The kind of investigation we have done here has got various applications in engineering and may have claims in practical problems, namely, Very Large Scale Integration of electronic chips, film coating etc. The richness of ideas and phenomena discussed in the proposed study can be expected to lead to highly productive interactions across disciplines.

Keywords: Thermocapillarity, Fluid Thin Film, Viscous Dissipation, Non-Darcy Medium, Surface Tension, Casson Fluid

Introduction: Fluid thin films can be seen in a variety of situations in nature and in various industrial applications viz. membrane in biophysics, tear films in eyes, coating flows, microfluidic engineering etc. The ubiquitous presence of thin films in nature and technology has led the scientists and researchers to address the mechanism involved in the flow. Despite the diversity of phenomena and applications, the mathematical model is quite similar if the fluid is sufficiently viscous. Reynolds (1886) was the first one who pointed out the need to model the dynamics of thin film flow, while studying the lubrication theory. Oron *et al.* (1997), in their review article, presented a unified mathematical theory for the macroscopic thin fluid films and discussed linear stability of the solution, considering different aspects of the problem. O'Brien and Schwartz (2002) considered the flow of semi-infinite uniform liquid layer film past a dry vertical substrate. Thiele et al. (2009) presented the analysis of the way in which thin fluid film evolve when the film was flowing down an inclined porous wall and discussed stability of the evolution equation and found that the substrate porosity tries to destabilize the flow. D'Alessio et al. (2010) studied the 2-dimensional flow of a thin fluid film, driven by gravity, down a wavy inclined surface which is being heated continuously. Flow of liquid thin films over stretching surfaces has garnered a lot of attention in current years because of their application in various industrial and engineering processes. Food stuff processing, designing of several heat exchangers, polymer processing, fiber and wire coating, Large Scale Integration (LSI) and Very Large Scale Integration (VLSI) of microchips are some of the examples where the thin liquid films over a stretching surface are apparent. Inspired by applications of such flows, a number of researchers devoted their time in analyzing the behavior of liquid thin films over a stretching surface, considering various characteristics of the problem. Some important research studies on the topic are due to Wang [6], Wang and Pop (2006), Dandapat et al. (2007), Liu and Anderson (2008), Santra and Dandapat (2009) and Liu and Megahed (2012).

In nuclear fusion power research, scientists use a device, known as divertor, which allows the removal of waste material from the plasma while the reactor is still operating. It has been observed that the thin liquid metal film, flowing fast enough may be a good option as a divertor surface for surface heat removal, under the hydromagnetic effects. Stimulated by such an application, Narula et al. (2003) led an experiment to examine the effects of magnetic field and its temporal and spatial gradients on the flow of thin liquid metal film. Hayat et al. (2008) observed in his study that the change in fluid velocity in regard to magnetic field is observed at the end of thin film whereas the magnetic field has almost no effect in central region of the film. Nadeem and Awais (2008) deliberated the influence of variable viscosity on the flow of thin liquid film over a shrinking sheet under the influence of magnetic field and analyzed the effects of pertinent flow parameters for transient as well as steady state cases. Dandapat et al. (2010) investigated the hydromagnetic thin film flow over a non-linearly stretching sheet and derived the evolution equation for film thickness. They concluded that the uniform film profile can only be obtained as a result of linear stretching, irrespective of the form of initial deposition. Das et al. (2015) considered the thermocapillarity effect on hydromagnetic thin film flow over an unsteady stretching sheet and found that the fluid temperature, skin friction and rate of heat transfer at sheet, all are adversely affected by thermocapillarity.

It is to be noted that, in all of the above investigations, the effect of viscous dissipation is not taken into account. The viscous dissipation is a phenomena of practical importance in many engineering devices. Although viscous dissipation effect is considered to be weak, but its effect becomes significantly important in surface tension driven flows e.g. tribology, in the instrumentation, food processing, lubrication, polymer manufacturing etc. Viscous dissipation changes the temperature distribution by playing a key role like an energy source which leads to affect heat transfer rate. Vajravelu and Hadjinicolaou (1993) analyzed heat transfer characteristics in the laminar boundary layer of a viscous and heat absorbing fluid over a stretching sheet taking viscous dissipation into account. Partha et al. (2005) studied the effect of viscous dissipation on the mixed convection flow with heat transfer from an exponentially stretching surface. Sanjayanand and Khan (2006) discussed heat and mass transfer in a viscoelastic boundary layer fluid flow over an exponentially stretching sheet taking viscous dissipation into account. Cortell (2008) investigated the effects of viscous dissipation and thermal radiation on the thermal boundary layer over a non-linearly stretching sheet. Aziz (2009) studied viscous dissipation effect on mixed convection flow of a micropolar fluid over an exponentially stretching sheet. Pavithra and Gireesha (2013) analyzed the effect of viscous dissipation on hydromagnetic fluid flow with heat transfer in a porous medium over an exponentially stretching sheet with fluid particle suspension.

As per the authors' concern, the surface tension driven magnetohydrodynamic flow of a Casson fluid thin film having variable thermal conductivity, past an unsteady stretching sheet, considering the viscous dissipation effect, in a non-Darcy porous medium, has not been considered yet. Although, the richness of ideas and phenomena discussed in the proposed study can be expected to lead to highly productive interactions across disciplines.

Mathematical analysis of the Problem: Consider the hydromagnetic flow of Casson fluid thin film through a non-Darcy porous medium, past an elastic sheet, which is being stretched unsteadily and emerging through a fine slit at the origin as depicted in Figure 1. The sheet is lying over $x-$ axis and $y-$ axis is taken in a direction perpendicular from the sheet. The flow region is permeated by a uniform magnetic field B_0, applied, in a transverse direction (i.e. parallel to y-axis). Furthermore, the effect of viscous dissipation is also taken into consideration. It is presumed that the flow is prompted due to elongation of the sheet by imposing a force, applied at one of the edges of the sheet in such a way that the sheet velocity is time dependent and varies linearly with the distance. Further, due to the thinness of liquid layer, the effect of buoyancy is also ignored, but it is not so thin that intermolecular forces come into play. The film thickness is assumed to be $h(t)$. The sheet starts stretching with a velocity $U(x,t)$, given as:

$$U(x,t) = \frac{cx}{1-\alpha t} \tag{1}$$

where α and c are constants (positive), having dimension of time^{-1}. It is to be observed that the sheet velocity given in (1) is valid only when $\alpha t < 1$ unless $\alpha = 0$. The sheet temperature is supposed to vary with *x* in a fashion, such as:

$$T_s = T_0 - T_{ref}\left(\frac{cx^2}{2\upsilon_0}\right)(1-\alpha t)^{-3/2} \tag{2}$$

where T_{ref} is reference temperature, T_0 is the temperature at the slit, *d* is an arbitrary constant, υ is kinematic coefficient of viscosity, *r* is space index and *m* is time index.

For incompressible and isotropic Casson fluid, the rheological equation of state is given as:

$$\tau_{ij} = \begin{cases} 2(\mu_\beta + P_y/\sqrt{2\phi})e_{ij}, & \phi > \phi_c, \\ 2(\mu_\beta + P_y/\sqrt{2\phi_c})e_{ij}, & \phi < \phi_c \end{cases} \tag{3}$$

where e_{ij} is the $(i, j)^{th}$ component of the deformation rate, τ_{ij} is the (i, j)th component of the stress tensor $\phi = e_{ij}e_{ij}$ is the product of deformation rate with itself, ϕ_c is the critical value of the ϕ based

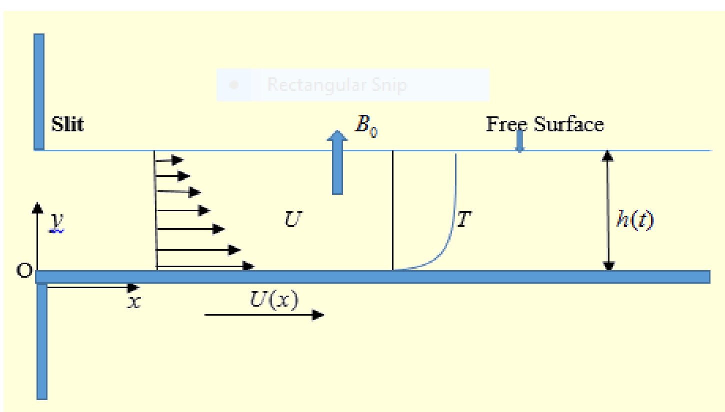

Figure 1: Geometry of the Problem

on the non-Newtonian model, μ_B is the plastic dynamic viscosity of non-Newtonian fluid and P_y is the yield stress of the fluid.

The variation in thermal conductivity and surface-tension with temperature are considered to be in the following form:

$$k = k_0[1 + b(T_s - T_0)] \tag{4}$$

$$\sigma = \sigma_0[1-\lambda(T-T_0)] \tag{5}$$

where k_0 and σ_0 are thermal conductivity and surface tension of fluid at slip temperature T_0. The constant λ is positive for the fluids such as air and water and it is negative for fluids such as lubrication oil.. For most of the liquids, surface tension is a decreasing function of temperature i.e. λ is positive for such fluids and *b* is a constant.

The fluid thin film is considered as non-volatile, therefore the influence of latent heat due to evaporation is ignored. The equations governing for fluid velocity and temperature field of the thin film, following the above mentioned behavior, are given as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon_0\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\varepsilon B_0^2}{\rho}u-\frac{\upsilon}{K}u-\frac{C_b}{\sqrt{K}}u^2 \tag{7}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\upsilon_0\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2 \tag{8}$$

where $\beta=\mu_b\sqrt{2\varphi_c}/P_y$ is Casson parameter, $u, v, T, \varepsilon, \sigma, \rho, k, c_p, K$ and C_b, are respectively, fluid velocity in *x*- direction, fluid velocity in *y*- direction, fluid temperature, electrical conductivity, variable surface tension, fluid density, variable thermal conductivity, specific heat at constant pressure, dynamic permeability of porous medium and quadratic drag coefficient.

It is worthwhile to observe that while writing the governing equations (6) to (8), induced magnetic field is ignored under the consideration of small magnetic Reynolds number. Since we are dealing with flow of a thin film over a sheet and thickness of the film, which in this case is characteristic length, is small and fluid velocity *u*, which evolve from the elongation of the sheet, is also small, thus the consideration of small magnetic Reynolds number is justified.

The conditions at the boundary are as following:

$$u=U(x,t),\ v=0,\ T=T_s \quad \text{at} \quad y=0 \tag{9}$$

$$\mu\frac{\partial u}{\partial y}=\frac{\partial \sigma}{\partial x},\quad \frac{\partial T}{\partial y}=0 \quad \text{at} \quad y=h \tag{10}$$

$$v=\frac{dh}{dt} \quad \text{at} \quad y=h \tag{11}$$

The Mathematical analysis of the flow of thin film can be made much simpler if we can convert the governing PDEs given by equations (6) to (8) along with the boundary conditions (9), (10) and (11) into ODEs. To do so, following similarity transformations are introduced:

$$\left.\begin{aligned} &\eta = \frac{1}{\gamma}\left(\frac{c}{\upsilon_0}\right)^{1/2}(1-\alpha t)^{-1/2} y,\ u = \frac{cx}{1-\alpha t} f'(\eta), v = \frac{-\sqrt{c\upsilon_0}}{(1-\alpha t)^{-1/2}} f(\eta), \\ &T = T_0 - T_{ref}\left(\frac{cx^2}{2\upsilon_0}\right)(1-\alpha t)^{-3/2}\theta(\eta),\ \theta(\eta) = \frac{T-T_0}{T_s - T_0}. \end{aligned}\right\} \tag{12}$$

These similarity transforms are valid only when $\alpha << 1/t$.

Making use of (12), equations (6), (7) and (8) get converted into following system of ordinary differential equations:

$$\left(1+\frac{1}{\beta}\right) f''' + ff'' - (1+F) f'^2 - S\left(\frac{\eta}{2} f'' + f'\right) - (M^2 + K_1) f' = 0 \tag{13}$$

$$\theta'' - \delta\,\theta\,\theta'' - \delta\,\theta'^2 + \left(1+\frac{1}{\beta}\right) E_c f''^2 - \Pr\left(2 f'\theta - f\theta' + S\left(\frac{\eta}{2}\theta' + 3\theta\right)\right) = 0 \tag{14}$$

Transformed boundary conditions are obtained as:

$$f = 0,\ f' = 1,\ \theta = 1, \quad \text{at } \eta = 0 \tag{15}$$

$$f'' = Ma.\theta,\ \theta' = 0 \quad \text{at } \eta = \gamma \tag{16}$$

$$f = \frac{S\gamma}{2} \quad \text{at } \eta = \gamma, \tag{17}$$

where $F = \left(C_b\, x / \sqrt{K}\right)$ is local inertia parameter, $M^2 = \varepsilon B_0^2 (1-\alpha t) / \rho c$ is magnetic parameter, $K_1 = \upsilon^2 \operatorname{Re}_x / KU^2$ is permeability parameter, $\operatorname{Re}_x = Ux/\upsilon$ is local Reynolds number, $\Pr = \rho\upsilon c_p / k$ is Prandtl number, $S = \alpha / c$ is unsteadiness parameter, $\delta = -b(T_s - T_0)$ is thermal conductivity parameter, $Ma = \dfrac{\lambda\sigma_0 T_{ref}\gamma}{\mu\sqrt{c\upsilon_0}}$ is thermocapillarity number and $E_c = U^2 / c_p(T_s - T_0)$ is Eckert number.

Here γ denotes the value of similarity transform η at the free surface. Thus the boundary condition (11) gives:

$$\gamma = (c/\upsilon_0)^{1/2}(1-\alpha t)^{-1/2} h(t) \tag{18}$$

Wall Velocity Gradient and Wall Temperature Gradient: For engineering and practical purposes, we are concerned with the exploration of physical entities of the flow behavior and heat transfer attributes by investigating the non-dimensional skin friction coefficient and local Nusselt number. These dimensionless parameters are presented as:

$$Cf_x = -\frac{2}{\gamma}\mathrm{Re}_x^{-1/2}\left(1+\frac{1}{\beta}\right)f''(0) \tag{19}$$

$$Nu_x = \frac{1}{2\gamma}\theta'(0)(1-\alpha t) \tag{20}$$

The Numerical Implementation: Differential equations (13) and (14), along with boundary conditions (15), (16) and (17) are solved numerically by shooting technique with 4th order Runge-Kutta method. Differential equations (13) and (14) are converted into a set of 5 first order linear differential equations. For an initial guess of γ, this set is solved, subject to conditions at the boundary (15) and (16) until the outer boundary condition (17) is satisfied.

Validation of the Numerical Solution: To judge the precision of numerical method, we have realized an assessment for the numerical values of skin friction coefficient $-f''(0)$ and γ with $M = E_c = F = K_1 = 0$ and $\beta \to \infty$, against *S* with those of Das et al. [16]. An excellent agreement is found between our result and that of Das et al. [16], which is shown in Table 1. This guarantees that our numerical scheme is in confidence and it can be used for further computation of results.

Table 1: Comparison of values of γ and $f''(0)$ for various values of *S* with published results for $M = E_c = F = K_1 = 0$ and $\mathrm{Pr} = 1,\ \beta \to \infty$

	Das et al. [16]		Present Study	
S	γ	$f''(0)$	γ	$f''(0)$
0.8	2.15199	2.680943	2.15198	2.68086
1.0	1.543626	1.972377	1.54360	1.972301
1.2	1.127780	1.444426	1.12780	1.443982
1.4	0.821055	1.012784	0.821032	1.012689
1.6	0.576173	0.642398	0.576170	0.642386

Results and Discussion: Extensive numerical computations are performed for velocity field and temperature distribution within the film boundary layer together with wall velocity gradient and wall temperature gradient, to get insight into the physics of the flow regime for several values of flow parameters which characterize the features of the flow. Numerical findings are well demonstrated in Figures 2 to 9 along with Table 2. The default values of pertinent flow parameters are taken as $F = 0.2, Ec = 0.4,\ \mathrm{Pr} = 1.3, M = 3, S = 1.2, Ma = 0.5,\ K_1 = 0.5,$ and $\beta = 0.5$, until otherwise specified particularly.

Figures 2 to 4 portrait the behavior of fluid velocity f' under the influence of magnetic field parameter M, Casson parameter β and thermocapillarity parameter Ma, respectively. We observe from Figure 2 that an increase in M causes fluid velocity inside the thin film as well as the film thickness to decrease significantly. The reason behind such effect of magnetic field is that since the existence of magnetic field in an electrically conducting thin film induces a retarding body force, referred to as Lorentz Force, and this force acts in a direction perpendicular to both fields. Since M suggests the ratio of hydromagnetic body force and viscous force, greater value of M indicates a stronger hydromagnetic body force which has a tendency to decelerate the fluid flow. Figure 3 is plotted to analyze the effect of Casson parameter β on fluid velocity and it is apparent from this figure that fluid velocity inside the thin film decreases on increasing Casson parameter β. Since Casson parameter β is directly proportional to the plastic dynamic viscosity μ_β. An increase in β means an increase in plastic dynamic viscosity μ_β, which means a greater resistance to the flow of thin film hence a downfall is observed in the fluid velocity on increasing the Casson parameter. Figures 4 elucidate that there is barely an effect of thermocapillarity parameter Ma on fluid thin film velocity, near the stretching sheet. However, as we get closer to free surface, the velocity is observed to increase on increasing the thermocapillarity parameter Ma.

Figures 5 to 9 are plotted to analyze the behavior of fluid temperature θ against β, M, E_c, δ and S. It is perceived from figure 5 and 6 that with an increase in β and M, temperature of thin film increases throughout the film region. Since the action of Casson parameter and magnetic field parameter has led to the decrement in the velocity of the fluid in thin film region, therefore, extra work done in dragging the fluid against these two physical entities, dissipates in the form of energy and hence increased fluid temperature is observed in the thermal boundary layer. Figure 7 elucidates that fluid temperature is getting decreased on increasing the Eckert number Ec, which in fact, supports the physics. It is perceived from Figure 8 that an increase in the thermal conductivity parameter δ causes an enhancement in the film temperature. Since an increase in thermal conductivity would obviously result in the enhancement of fluid temperature. Figure 9 shows the effect of unsteadiness parameter S on the film temperature. One can observe from this figure that an increase in unsteadiness in the stretching results in the decreased fluid temperature inside the thin film and reduced film thickness.

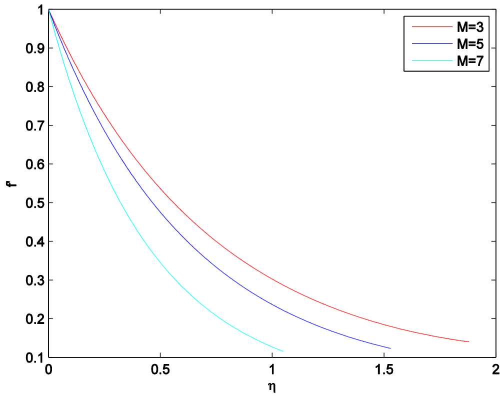

Fig. 2: Velocity Profile for M

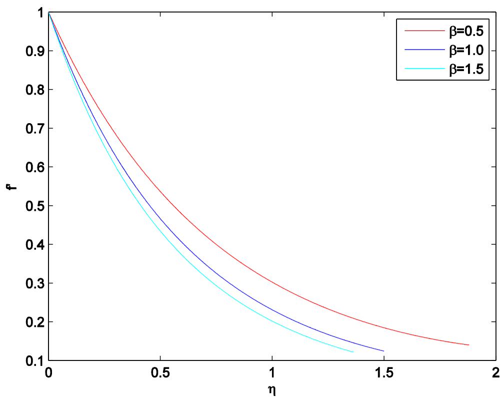

Fig. 3: Velocity Profile for β.

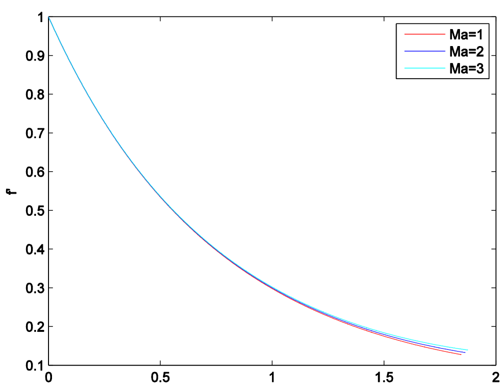

Fig. 4: Velocity Profile for *Ma*

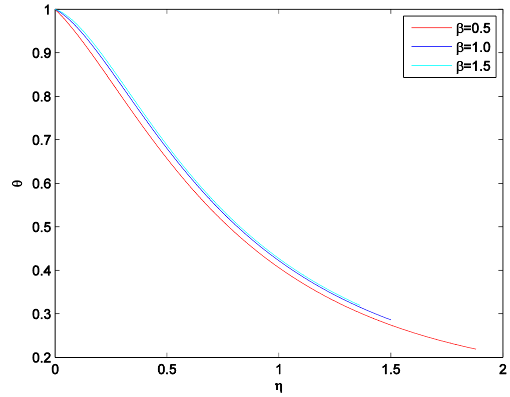

Fig. 5: Temperature Profile for β.

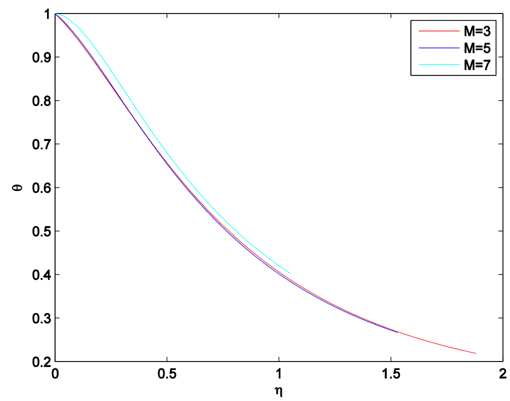

Fig. 6: Temperature Profile for *M*.

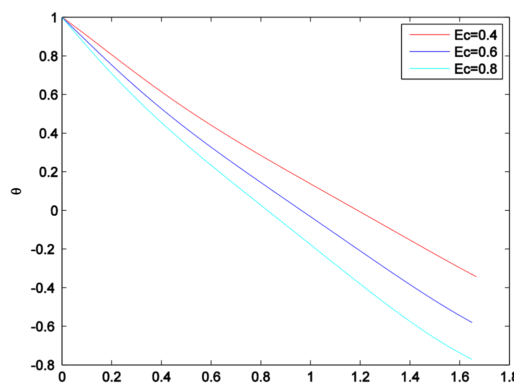

Fig. 7: Temperature Profile for *Ec*

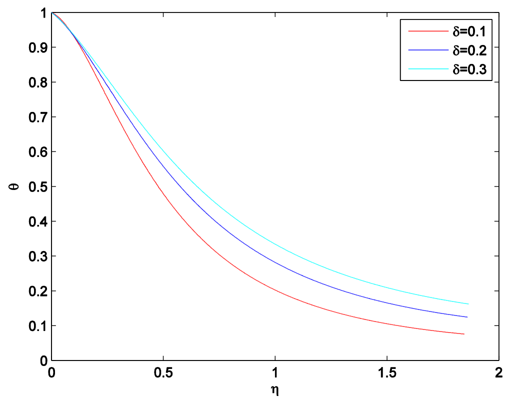

Fig. 8: Temperature Profile for δ.

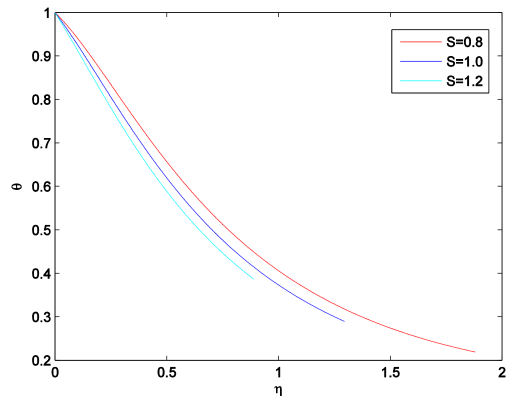

Fig. 9: Temperature Profile for S

In order to analyze the behavior of physical quantities of interest viz. local skin friction co-efficient $\frac{1}{2}Cf_x \mathrm{Re}_x^{1/2}$ and local Nusselt number $Nu_x \mathrm{Re}_x^{-1/2}$, effects of pertinent flow parameters such as β, S, M, Ma and E_c, on these two quantities are computed and are presented in Table 2. One can observe from Table 2 that skin friction co-efficient is getting enhanced on increasing either of Casson parameter, unsteadiness parameter, Magnetic field parameter and thermocapillarity number. On the other hand, local Nusselt number is getting enhanced on increasing only unsteadiness parameter and decreases on increasing either of Casson parameter, Magnetic field parameter, thermocapillarity number and Eckert number.

Table 2: Skin friction C_{f_x} (i.e. $-f''(0)$) coefficient and Nusselt number N_u (i.e. $\theta'(0)$) for various values of flow parameters

β	S	M	Ma	E_c	$f''(0)$	$\theta'(0)$
0.5	0.8	3	1	0.4	-1.2898	-0.4944
1.0	0.8	3	1	0.4	-1.5850	-0.2649
1.5	0.8	3	1	0.4	-1.7373	-0.1650
0.5	**0.8**	3	1	0.4	-1.2898	-0.4944
0.5	**1.0**	3	1	0.4	-1.3053	-0.6672
0.5	**1.2**	3	1	0.4	-1.3202	-0.8195
0.5	0.8	**3**	1	0.4	-1.2898	-0.4944
0.5	0.8	**5**	1	0.4	-1.5287	-0.4185
0.5	0.8	**7**	1	0.4	-1.7340	-0.3631
0.5	0.8	3	**1**	0.4	-1.2898	-0.4944
0.5	0.8	3	**2**	0.4	-1.2917	-0.2826
0.5	0.8	3	**3**	0.4	-1.2932	-0.2803
0.5	0.8	3	1	**0.4**	-1.2898	-0.4944
0.5	0.8	3	1	**0.6**	-1.3160	-1.2339
0.5	0.8	3	1	**0.8**	-1.3152	-1.5246

Conclusions:

A mathematical model is established for surface tension driven magnetohydrodynamic non-Darcy flow of an incompressible, electrically conducting and viscous thin fluid film past a horizontal unsteady stretching sheet with variable thermal conductivity, taking viscous dissipation effect into consideration. Noteworthy results are summarized as follows:

- An intensification in magnetic field leads to a significant fall in fluid velocity and reduces the film thickness as well. A greater resistance to the flow is offered on increasing the plastic dynamic viscosity due to virtue of increasing Casson parameter. On the other hand, thermocapillarity tends to increase the film velocity in the region near the free surface.
- As a result of increased resistance to the fluid movement due to increase in Casson and magnetic parameters, the fluid temperature is getting increased. A decrease in fluid temperature is also witnessed on increasing unsteadiness parameter. However, o increasing the thermal conductivity, temperature is observed to increase for the obvious reasons.
- Coefficient of skin friction is getting boosted on increasing either of Casson parameter, unsteadiness parameter, Magnetic field parameter and thermocapillarity parameter. On the other hand, local Nusselt number is perceived to rise only on increasing unsteadiness parameters while the rest of the parameter, namely Casson parameter, Magnetic field parameter, thermocapillarity number and Eckert number have adverse effect on local Nusselt number.

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