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UNDERSTANDING THE ROLE OF AXIAL WALL CONDUCTION IN MINI-/MICRO-COUNTERFLOW HEAT EXCHANGERS

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ABSTRACT The current trend towards miniaturization of electro-mechanical systems (MEMS) has boosted the development of new applications in the field of mini- and micro-heat exchangers. The accompanying growth of the wall-to-channel dimensional ratios is known to increase the role of axial wall conduction, which reduces the effectiveness of counterflow heat exchangers to that of coflow systems in the limit of large axial wall conduction. Understanding the impact of wall conduction on the performance of mini- and micro-heat exchangers is thus critical for the design of high efficient equipment. In this work we investigate the role of axial and transverse wall conduction by consideration of a simple problem, the laminar counterflow parallel-plate heat exchanger, which has been recently used by the authors as a cartoon to represent the heat exchange process in real-life counterflow heat exchangers. Using as starting point the exact series solution of Quintero & Vera [2017], we carry out a thorough parametric study discussing the effect of the two main dimensionless parameters determining the effect of multidimensional wall conduction on heat exchanger performance; namely, the dimensionless wall thermal resistance, κ_w^{-1}, and the dimensionless wall thickness, Δ_w. Isocontours of heat exchanger effectiveness in the (Δ_w, κ_w^{-1}) plane exhibit well-defined asymptotic regimes for limiting values of the parameters corresponding to cases with/withouth significant axial/transverse wall conduction, with smooth transitions between the different regimes ocurrying for values of Δ_w and κ_w^{-1} of order unity. The analysis provides accurate conditions for neglecting axial and transverse wall conduction effects, and shows that the optimum wall conductivity, previously discussed in the literature, ceases to exist for moderately thick walls.

PROBLEM FORMULATION

General Problem The general non-dimensional problem stated by Quintero & Vera [2017] includes the energy equation for the fluids, without consideration of axial heat conduction,

$$\frac{3}{4}(1-y_1^2)\frac{\partial \theta_1}{\partial \xi} = \frac{\partial^2 \theta_1}{\partial y_1^2} \qquad [1]$$

$$-m\frac{3}{4}(1-y_2^2)\frac{\partial \theta_2}{\partial \xi} = \frac{\partial^2 \theta_2}{\partial y_2^2} \qquad [2]$$

in $0 < \xi < \xi_L$, $0 < y_i < 1$, and the heat equation for the wall

$$\Delta_{\mathrm{w}}^2 \frac{\partial^2 \theta_{\mathrm{w}}}{\partial \xi^2} + \frac{\partial^2 \theta_{\mathrm{w}}}{\partial y_{\mathrm{w}}^2} = 0 \qquad [3]$$

in $0 < \xi < \xi_L$, $-1/2 < y_{\mathrm{w}} < 1/2$. The above equations have to be integrated with the no-flux boundary conditions at the channel symmetry planes $\partial\theta_1/\partial y_1 = \partial\theta_2/\partial y_2 = 0$ at $0 < \xi < \xi_L$, $y_i = 0$, and with the compatibility conditions of continuity of temperatures

$$\begin{aligned} \theta_1(\xi, 1) &= \theta_{\mathrm{w}}\left(\xi, -{}^1\!/_2\right) \equiv \theta_{\mathrm{w}1}(\xi) \\ \theta_2(\xi, 1) &= \theta_{\mathrm{w}}\left(\xi, {}^1\!/_2\right) \equiv \theta_{\mathrm{w}2}(\xi) \end{aligned} \quad \text{at} \quad 0 < \xi < \xi_L \qquad [4]$$

and wall-normal heat fluxes

$$\begin{aligned} \left.\frac{\partial\theta_1}{\partial y_1}\right|_{y_1=1} &= \kappa_{\mathrm{w}} \left.\frac{\partial\theta_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\right|_{y_{\mathrm{w}}=-1/2} \equiv \nu_1(\xi) \\ -k\left.\frac{\partial\theta_2}{\partial y_2}\right|_{y_2=1} &= \kappa_{\mathrm{w}} \left.\frac{\partial\theta_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\right|_{y_{\mathrm{w}}=1/2} \equiv \nu_2(\xi) \end{aligned} \quad \text{at} \quad 0 < \xi < \xi_L \qquad [5]$$

at the fluid-wall interfaces. Uniform temperature profiles are specified at the inlet sections of both fluids, $\theta_1 = \theta_{1,\mathrm{in}} = 0$ at $\xi = 0$, $0 \le y_1 < 1$, and $\theta_2 = \theta_{2,\mathrm{in}} = 1$ at $\xi = \xi_L$, $0 \le y_2 < 1$, while adiabatic boundary conditions are imposed at both ends of the heat exchanging wall, $\partial\theta_{\mathrm{w}}/\partial\xi = 0$ at $\xi = 0$, $-1/2 < y_{\mathrm{w}} < 1/2$, and $\partial\theta_{\mathrm{w}}/\partial\xi = 0$ at $\xi = \xi_L$, $-1/2 < y_{\mathrm{w}} < 1/2$. The exact solution for the temperature field can be written in the form of an eigenfunction expansion $\theta_i(\xi, y_i) = A + \sum_{n=-\infty}^{+\infty} C_n e^{-\lambda_n \xi} h_{n,i}(y_i)$, $i = 1, 2, w$, with the eigenvalues, λ_n, eigenfunctions, $h_{n,i}(y_i)$, and constants, A and C_n, obtained as shown by Quintero & Vera [2017].

Simplified Problems Conditions can be established to determine whether the longitudinal and transverse wall conduction effects are relevant for the calculations or not. In the resulting simplified problems, without significant effects of axial and/or transverse wall conduction, the thermal problem for the fluids remains the same, whereas the thermal problem for the wall admits different simplifications.

<u>*Negligible axial wall conduction*</u> In the limit $\Delta_{\mathrm{w}} \ll 1$, axial wall conduction can be safely neglected and the heat equation, Eq. [3], becomes

$$\frac{\partial^2 \theta_{\mathrm{w}}}{\partial y_{\mathrm{w}}^2} = 0 \qquad [6]$$

Due to the absence of the axial conduction term, the above equation can be readily integrated once to yield a constant heat flux across the wall, so that the heat released to the wall by fluid 2 at a given downstream station ξ is absorbed by fluid 1 at the same location. The linear temperature gradient that appears in the wall transverse direction results in a finite wall temperature jump, so that the coupling conditions stated by Eqs. [4]-[5][5] can be rewritten as

$$\begin{aligned} \theta_2 - \theta_1 &= \frac{1}{\kappa_{\mathrm{w}}} \frac{\partial\theta_1}{\partial y_1} \equiv \frac{\nu(\xi)}{\kappa_{\mathrm{w}}} \\ \frac{\partial\theta_1}{\partial y_1} &= -k\frac{\partial\theta_2}{\partial y_2} \equiv \nu(\xi) \end{aligned} \quad \text{at} \quad 0 < \xi < \xi_L, \quad y_i = 1 \qquad [7]$$

The absence of longitudinal heat conduction in the fluids implies that at both ends of the heat exchange region the interfacial wall temperature is imposed by the fluid entering the heat exchanger in that section, hence $\theta_{\mathrm{w}}(0, -1/2) = \theta_1(0,1) = 0$ and $\theta_{\mathrm{w}}(\xi_L, 1/2) = \theta_2(\xi_L, 1) = 1$.

Based on the axial conduction number concept stated by Maranzana et al. [2004], Quintero [2016] derived a generalized criterion to determine the conditions under which axial wall conduction can be neglected

$$\kappa_{\mathrm{w}}\Delta_{\mathrm{w}}^2 < 0.05\left(\xi_L + \left[\frac{17}{70}\frac{k+1}{k} + \frac{1}{2\kappa_{\mathrm{w}}}\right]\right) \qquad [8]$$

The problem corresponding to the limit $\Delta_{\mathrm{w}} \ll 1$ was solved analytically by Quintero et al. [2014].

<u>*Thermally thin walls*</u> For sufficiently high thermal wall conductivity, the temperature jump across the wall can be neglected and the coupling boundary conditions at the fluid-wall interface stated in Eq. [7] reduced

$$\begin{aligned} \theta_2 = \theta_1 \equiv \theta_{\mathrm{w}}(\xi) \\ \frac{\partial \theta_1}{\partial y_1} = -k\frac{\partial \theta_2}{\partial y_2} \equiv \nu(\xi) \end{aligned} \quad \text{at} \quad 0 < \xi < \xi_L, \quad y_i = 1 \qquad [9]$$

Quintero [2016] proposed the following criterion to neglect the wall temperature jump

$$\kappa_{\mathrm{w}}^{-1} < 0.02\left(\xi_L + \left[\frac{17}{70}\frac{k+1}{k}\right]\right) \qquad [10]$$

The problem corresponding to the double limit $\Delta_{\mathrm{w}} \ll 1$ and $\kappa_{\mathrm{w}}^{-1} \ll 1$ was solved analytically by Vera & Liñán [2010].

RESULTS AND CONCLUSIONS

The central panels of Fig. 1 show contour plots of heat exchanger effectiveness, ε, normalized with the maximum effectiveness, $\varepsilon^{\max}$, reached in all cases in the limit of thermally thin walls without axial heat conduction, i.e., on the lower left corner of the plots. The lower central panel shows the four regions into which the $(\Delta_{\mathrm{w}} - \kappa_{\mathrm{w}}^{-1})$ plane can be divided according to Eqs. [8]-[10]. When axial wall conduction is relevant in the performance of the heat exchanger (regions 3 and 3'), an optimum dimensionless wall thermal resistance exists for a given dimensionless wall thickness which maximizes the effectiveness, as previously stated in other investigations (Mori et al. [1980], Maranzana et al. [2004]). For short heat exchangers (upper central panel) the optimum dimensionless wall thermal resistance ceases to exist. In this case the heat is transferred from fluid 2 to the wall at the thermal entry region of fluid 2, then it travels longitudinally along the wall, until it is released to fluid 1 in the thermal entry region of fluid 1. Thus, even though the heat exchanger operates in counterflow configuration, its performance is similar to that of a coflow system.

The left and right panels of Fig. 1 show, respectively, the bulk temperature of fluids 1 and 2 and the interfacial fluid-wall temperature profiles, and the interfacial and longitudinal wall heat fluxes. Plots indicated with the label "A" illustrate a case with finite wall temperature jump but negligible axial wall conduction, where the heat released at a specific downstream station from fluid 2 to the wall is entirely transferred to fluid 1 at that same location. Instead, plots indicated with the label "B" illustrate a case with significant effects of axial wall conduction but without wall temperature jump. Note that in this case the emperature profiles (lower left panel) are similar to those obtained in coflow configurations, and the heat flux profiles (lower right panel) show that a significant fraction of the heat transferred by fluid 2 to the wall is conducted along the wall instead of been released to fluid 1.

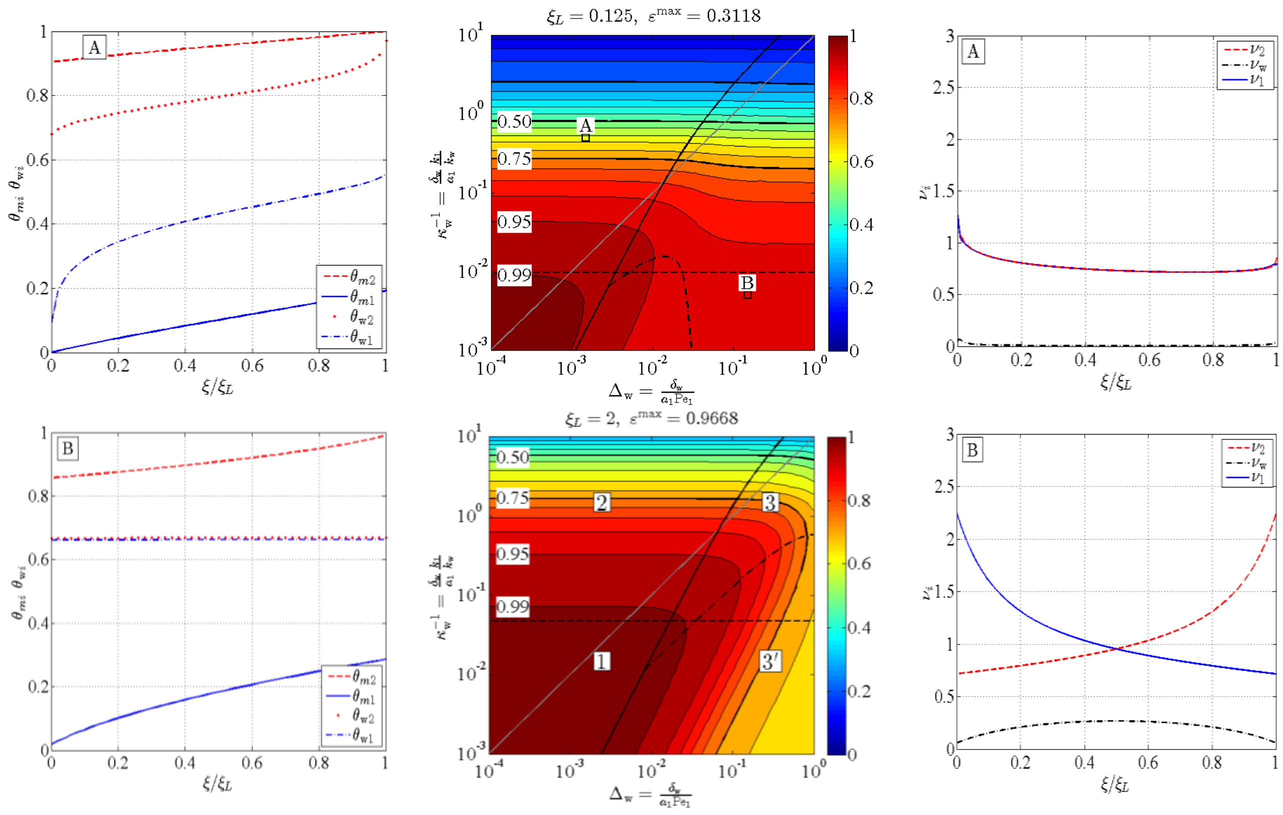

Figure 1. Left: mean bulk fluid temperatures (solid and dashed lines), and interfacial fluid-wall temperatures (dot-dashed lines) for a heat exchanger of dimensionless length $\xi_L = 0.125$, dimensionless wall thickness $\Delta_w = [0.0015, 0.15]$, and dimensionless wall thermal resistance $\kappa_w = [2, 200]$. Center panels: normalized heat exchanger effectiveness isocontours in the $(\Delta_w, {\kappa_w}^{-1})$ plane for a short heat exchanger, $\xi_L = 0.125$, and a long heat exchanger, $\xi_L = 2$; the solid line shows condition Eq. [8], while the dashed line shows condition Eq. [10]. Right panels: heat fluxes across the fluid-wall interfaces (solid and dashed lines), and longitudinal heat flux across the wall (dot-dashed line) for $\xi_L = 0.125$, $\Delta_w = [0.0015, 0.15]$, and $\kappa_w = [2, 200]$. Results for $m = 1$ and $k = 2$.

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