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ASPECT RATIO EFFECTS ON LAMINAR RAYLEIGH-BÉNARD CONVECTION OF POWER-LAW FLUIDS IN RECTANGULAR ENCLOSURES: A NUMERICAL INVESTIGATION

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Natural convection of inelastic non-Newtonian fluids obeying the power-law model of viscosity ($\mu = K\dot{\gamma}^{n-1}$ where μ is the viscosity, K the consistency index, n the power-law index and $\dot{\gamma}$ the shear rate) in rectangular enclosures with differentially heated horizontal walls with heated bottom wall (i.e. Rayleigh-Bénard configuration) has been analysed. Two-dimensional numerical simulations for various aspect ratios AR ($= H/L$, where H is enclosure height and L is the enclosure length) ranging from 1/8 to 8 for a range of different values of power-law exponents (i.e. $n = 0.6 - 1.8$) and nominal Rayleigh number (i.e. $Ra = g\beta(T_H - T_C)H^{2n+1}\alpha^{-n}(K/\rho)^{-1} = 10^3 - 10^5$, where g, β, T_H, T_C, α and ρ are the acceleration due to gravity, volume expansion coefficient, hot wall temperature, cold wall temperature and density respectively). The analysis in this paper has been carried out for a single value of nominal Prandtl number (i.e. $Pr = \alpha^{n-2}H^{2-2n}(K/\rho)$=1000), typical of viscous non-Newtonian fluids, because several previous analyses (Turan *et al.* [2013], Turan *et al.* [2014]) demonstrated that the mean Nusselt number and flow pattern for natural convection of power-law fluids remain insensitive to the change in nominal Prandtl number. For the current analysis the differentially heated horizontal walls are considered to be subjected to constant wall temperatures. The numerical simulations have been carried out in the framework of the finite-volume technique where the non-dimensional conservation equations of mass, momentum and energy are solved in a coupled manner using a semi-implicit pressure based SIMPLE algorithm (Patankar [1980]). The simulations have been conducted on different sized grids to ensure grid convergence and two-dimensional simulations are used to carry out an extensive parametric sweep encompassing different values of Ra, AR and n. The simulation results are bench-marked with respect to previous findings by de-Vahl Davis [1983] and Kim *et al.* [2003] for Newtonian and power-law fluids respectively.

It has been found that convection weakens with increasing aspect ratio and the heat transfer takes place purely due to thermal conduction for tall enclosures (i.e. $AR >> 1$ apart from $AR = 2$ for $Ra > 5\times10^4$) for all values of Ra and n. It will be demonstrated in the paper based on scaling arguments that the relative contribution of advection (diffusion) to overall thermal transport weakens (strengthens) with increasing AR for a given set of values of Ra, n and *Pr*. Additionally, the flow pattern for $AR \leq 1$ has been found to be dependent not only on Ra and n but also on the choice of initial condition used for the simulation. Although viscous resistance weakens with decreasing power-law exponent for a given set of values of Ra, AR and *Pr*, the mean Nusselt number $\overline{Nu} = h_m H/k$ (where h_m is the mean heat transfer coefficient and k is thermal conductivity) does not show a monotonic increase with decreasing n for $AR \leq 1$ because of the change in flow pattern (i.e. number of convection rolls/cells). Such effects are illustrated for $AR = 0.5$ in Fig. 1 in this abstract. Moreover, it has been found that the flow pattern and the mean Nusselt number $\overline{Nu}$ are dependent on initial conditions and it is possible to obtain different steady-state solutions for different initial conditions. This non-uniqueness of flow and heat transfer patterns is prevalent for Newtonian (i.e. $n = 1$) and shear-thinning (i.e. $n < 1$) fluids. Furthermore, it is possible to obtain a steady solution for shear-thinning (i.e. $n < 1$) fluids for some initial conditions, whereas other initial conditions yield unsteady flow patterns.

A detailed scaling analysis is carried out for buoyancy-driven power-law fluid flows, which reveals that the effective Rayleigh number Ra_{eff} increases with decreasing (increasing) n and AR (nominal Rayleigh number Ra) for a given value of Ra (AR and n). The effective Rayleigh number Ra_{eff} may assume such a large value for small values of n and/or AR that it may not be possible to obtain a steady solution. The nominal Rayleigh number Ra above which a steady solution cannot be obtained decreases with decreasing n and AR for a given value of Ra. The aspect ratio AR and power-law exponent n also affect the critical Rayleigh number Ra_{crit} for the onset of convection and Ra_{crit} has been found to decrease with decreasing n and AR. The scaling analysis will be utilised to provide detailed physical explanations for the aforementioned behaviour.

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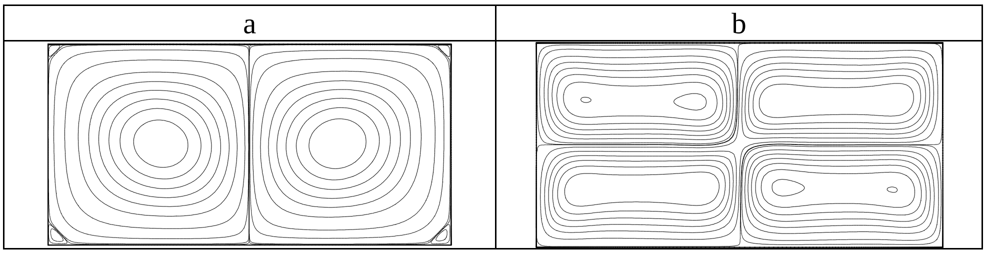

Ra_H \ n	$\overline{Nu}$						
	0.6	0.8	1	1.2	1.4	1.6	1.8
10^3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5×10^3	3.886-a	2.693-a	1.906-a	1.262-a	1.028-a	1.006-a	1.003-a
10^4	4.778-a	3.273-a	2.403-a	1.762-a	1.262-a	1.057-a	1.017-a
5×10^4	4.550-b	4.943-a	3.567-a	2.734-a	2.157-a	1.699-a	1.345-a
10^5	No steady solution	5.855-a	4.165-a	3.169-a	2.512-a	2.026-a	1.633-a

Fig. 1: Flow patterns and mean Nusselt number for different values of Ra_H and n for AR=0.5.