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The Elliptic Emmons Problem

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Abstract

The Emmons' problem is concerned with burning of a flat fuel surface in an oxidizer stream flowing parallel to the surface. The primary objective is to determine the steady burning rate of a vaporizing liquid (or a subliming solid) that supports the flame in the boundary layer. This classical problem provides a well-defined geometry with analytical solutions that is relatively easy to establish experimentally. It has therefore been very useful for flammability assessment of materials. The similarity solution with the boundary layer approximation was obtained by Emmons [1]. In this paper, the Emmons' problem is reformulated as a solution of the full Navier-Stokes equations without invoking the boundary layer approximation. Exact analytical solutions are developed for the mass and mixture fraction conservation equations in parabolic coordinates. The corresponding velocity field incorporates both the Emmons boundary layer result and an elliptic upstream influence that asymptotically satisfies the full Navier-Stokes equations. It also includes a solution for the pressure perturbation. The solutions for the velocity and pressure fields are exact everywhere except in a small region O (20 Stokes lengths ~ 2mm) surrounding the leading edge. The size of this region is comparable to a diffusion flame thickness, a quantity that is approximated as a line in flame sheet approximation. However, the singularity at the leading edge is geometrical, and unlike the boundary layer solution, the singularity is confined to a point rather than the whole line x=0. This framework is used to analyze soot transport with generation and destruction. The soot model is also analytically tractable and seems to yield physically plausible results. Since only the general form of the model can be prescribed a priori, the physical constants of the soot model will have to be determined experimentally. The results are of interest both as a contribution to laminar combustion theory and as a test case for CFD simulation codes.

[1] Emmons, H.W., "The Film Combustion of Liquid Fuel", Z. Angew. Math. Mech., Vol. 36, p. 60 (1956).