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NONLINEAR COMPUTATION: MULTIPLICITY AND STABILITY

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The term “Nonlinear Computation” is a short-form for “computation of something that is nonlinear”. It provides a framework for the computational science and engineering of the future. Topics such as bifurcation and chaos, methods such as continuation and branch switching, and features such as stability and sensitivity are basic ingredients in the attempt to understand our world. Nonlinear Computation is essential when we try to predict and simulate the dynamics of states in the biological systems we belong, in the ecological and economical systems we live, and in the technical systems we utilize.

We will fist discuss key issues in Nonlinear Computation and demonstrate its capabilities by using examples such as (1) cooling systems of rotating machinery, (2) fabrication and manipulation of nanobubbles, nanodroplets and nanofluids, (3) thermal control system of the Alpha Magnetic Spectrometer (AMS) on the International Space Station, and (4) causative factors and the clinical applicability of spontaneous regression of malignant tumors, and then detail some rich features of multiplicity and stability by using forced convection in microchannels as an example which are summarized in the rest of this abstract.

In an attempt to quantify the effects of channel curvature, initial conditions and disturbances, a numerical study is made on the fully-developed bifurcation structure and stability of the forced convection in curved microchannels of square cross-section. The governing differential equations from the conservation laws are discretized by the finite volume method to obtain discretization equations, a set of nonlinear algebraic equations. The discretization equations are solved for parameter-dependence of flow and temperature fields by the Euler-Newton continuation with the solution branches parameterized by the pseudo Dean number, the arclength or the local variable. The bifurcation points are detected by the test function. The Hopf bifurcation point is determined by the transient computation. The branch switching is made by a scheme approximating the difference between branches. Two symmetric and eight asymmetric solution branches are found with eight symmetry-breaking bifurcation points and thirty-one limit points.

The dynamic responses of multiple steady flows to the 2D finite random disturbances are examined by the direct transient computation. The flows are found to respond to three sets of finite random disturbances in the same manner in the range $0 \le De \le 357.77$. Here *De* is the Dean number defined by $De = \mathrm{Re}\sqrt{\sigma}$ (*Re* is the Reynolds number, σ the curvature ratio). At any fixed value of Dean number in the range $0 \le De \le 313.05$, all steady flows develop, after the initial finite random disturbances, to the same final state. The finite random disturbances lead the multiple steady flows to a stable steady 2-cell state in $0 \le De \le 128.09$, a temporal periodic oscillation between symmetric/asymmetric 2-cell flows and symmetric/asymmetric 4-cell flows in

$128.09 < De \leq 219.13$, another stable steady 2-cell state in $219.13 < De \leq 313.05$, a temporal oscillation with the intermittency in $313.05 < De \leq 355.77$, and a chaotic oscillation in *De* > 357.77. The intermittent flow is characterized by the flow oscillation among the symmetric/asymmetric 2-cell patterns during the period between two bursts and among the symmetric/asymmetric 4-cell structures during the burst. The chaotic flow is, on the other hand, featured by the flow oscillation among the 4-cell patterns during the burst, and the 2-cell structures during the other period.

Both the mean friction factor and the mean Nusselt number are obtained with their correlating relations listed for all physically-realizable fully-developed flows. The channel curvature can enhance the heat transfer significantly at the expense of slight increase of flow resistance. The drastic change of mean Nusselt number occurs when flows shift between the stable steady flows and the temporal oscillating flows.

The channel curvature, the flow multiplicity and the stability are at least partially responsible for the large differences in the reported friction factors and heat transfer coefficients in the literature, and should be further studied in order to provide the tools necessary for optimal design and process control of various MEMS and modern instruments used in chemical analysis and biomedical diagnostics. New functional microdevices can also be produced by creatively using or tailoring multiple states of flows and heat transfer in microchannels.