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RAYLEIGHT-BENARD CONVECTION IN A HORIZONTAL POROUS CAVITY SATURATED BY A SHEAR-THINNING FLUID

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ABSTRACT Shear-thinning fluids flows coupled with heat transfer are encountered in most of industrial applications related to chemical, petrochemical, biomedical and molding processes. Such fluids, with complex and heavy molecular structure, do not obey the classical law of a Newtonian fluid, where the viscosity remains constant regardless the shear rate. Examples of such fluids are polymer solutions, blood, magma, crude oils fluids, to name but a few. In the literature, most of thermal convection investigations in porous media dealt with Newtonian fluids or non-Newtonian fluids with simple rheological behavior, such as power-law fluids valid for relatively high shear rate. A little attention was paid to heat transfer in shear-thinning fluids with complex rheological behavior ranging from zero to infinity shear rates.
The present study is focused on Rayleigh-Benard convection in an isotropic porous media saturated with a non-Newtonian shear thinning fluid with a rheological behavior obeying to the general model developed by Carreau-Yasuda. The Rayleigh-Benard convection configuration consists of a shallow porous cavity, as displayed in Figure 1, saturated with a shear-thinning fluid and subject to a vertical temperature gradient. The layer is characterized by the aspect ratio *A=L/H*, where *H* is the height and *L* is the length of the layer. The flow in the porous media is modeled using the Darcy law and the Boussinesq approximations. The effect of the apparent viscosity of the fluid and its rheological parameters on the onset of convection is investigated.

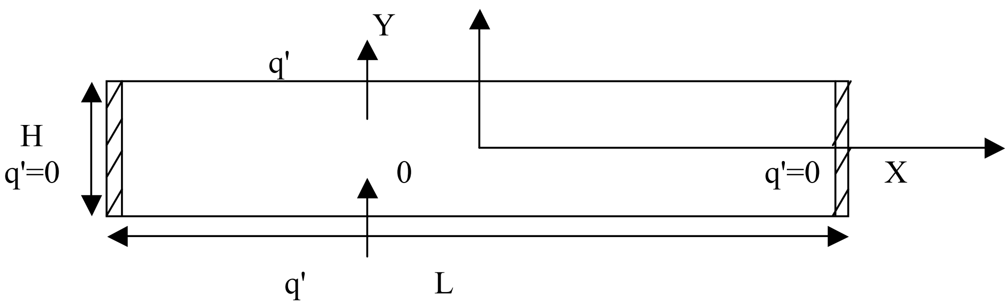

Fig 1. Sketch of the geometry and coordinates system

The dimensionless governing equations are given below using the stream function formulation. The equation of motion based on Darcy law reads:

$$\nabla^2\psi = -\frac{1}{\square}\left[Ra\left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial \square}{\partial y}\frac{\partial \psi}{\partial y}\right) + \left(\frac{\partial \square}{\partial x}\frac{\partial \psi}{\partial x}\right)\right] \quad (1)$$

and the energy equation is given as:

$$\frac{\partial T}{\partial t} + \left(u\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y}\right) = \nabla^2 \mathrm{T} \tag{2}$$

Where ψ is the dimensionless stream function, T the dimensionless temperature, u and v the dimensionless velocity components, t the dimensionless time and x and y are the dimensionless coordinates axes, and Ra is the Darcy-Rayleigh number based on the zero-shear rate fluid viscosity:

$$Ra = \frac{K\rho_0 g\beta_T\, \Delta T^{'} H'}{\mu_0\, \alpha}$$

porous matrix permeability, $\Delta T^{'}$ the temperature characteristic and $\square$is the apparent viscosity of the fluid, which depends on the shear rate,$\dot{\gamma}$.The parameter $\square$ is defined, in its general form, following the Carreau-Yasuda model [1]-[2]:

$$\square(\dot{\gamma}) = s + (1 - s)[1 + (E\dot{\gamma})^a]^{\frac{n-1}{a}} \tag{4}$$

where

$$\square = \frac{\mu}{\mu_0}, \qquad E = \frac{E'\square}{\varepsilon H'^2}, \qquad s = \frac{\mu_0}{\mu_\infty}$$

and μ is the fluid viscosity; μ_0 is the fluid viscosity at zero shear rate and μ_∞ is the viscosity at infinite shear rate. For the rheological parameters, E is the dimensionless elasticity (time), $\dot{\gamma}$ is the magnitude of the shear rate, n the power-law exponent, α the effective thermal diffusivity andε the porosity of the porous matrix. The convective flow inside the porous layer is governed by the following controlling parameters: the Rayleigh, Ra, the viscosities ratio, s, the power law exponent, n, the elasticity E, and the exponent a.

The shear rate is defined as follows:

$$\dot{\gamma} = \sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2}$$

The boundary conditions are given by:

$$x = \pm\frac{A}{2} \;:\; \psi = 0 \qquad\qquad and \quad y = \pm\frac{1}{2} : \psi = 0 \tag{5}$$

$$x = \pm\frac{A}{2} \;:\; \frac{\partial T}{\partial x} = 0 \qquad\qquad and \quad y = \pm\frac{1}{2} : \frac{\partial T}{\partial y} = -1 \tag{6}$$

The heat transfer rate is expressed in terms of the Nusselt number defined as:

$$Nu = \frac{\Delta T'}{\overline{\Delta T'}} = \frac{1}{\Delta T} = \frac{1}{(T_{(0,-0.5)} - T_{(0,+0.5)})} \qquad (7)$$

where ΔT is the dimensionless temperature difference across the section.

The above differential equations are non-linear and solved numerically using a finite difference method. In the final paper, The effects of the governing parameters on the convective flow and the heat transfer rate will be investigated.

REFERENCES

[1] P. Carreau. " Reological equation from molecular network theories", Ph.D. Thesis, University of Wisconsin, Madison (1968).

[2] K.Yasuda. "Investigation of the analogies between viscometric and lineare viscoelastic properties of polystyrene", Ph.D.Thesis, MIT, Cambridge.

Keywords*: Rayleigh–Benard Convection, rectangular cavity, finite difference method, shear-thinning fluid, porous media.*