

## **A THERMODYNAMICALLY CONSISTENT PHASE-FIELD MODEL FOR TWO-PHASE FLOWS AND ITS COMPUTATIONS**

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### **ABSTRACT**

We develop a phase-field model for binary incompressible fluid with thermocapillary effects, which allows for different properties (densities, viscosities and heat conductivities) of each component while maintaining thermodynamic consistency. A sharp-interface limit analysis is carried out to show that the interfacial conditions of the classical sharp-interface models can be recovered from our phase-field model. Numerical examples computed through a continuous finite element method show that the results of the model are consistent with some existing analytical and numerical results.

### **EXTENDED ABSTRACT**

We develop a phase-field model for binary incompressible (quasi-incompressible) fluid with thermocapillary effects, which allows for the different properties (densities, viscosities and heat conductivities) of each component while maintaining thermodynamic consistency. The governing equations of the model including the Navier-Stokes equations with additional stress terms, Cahn-Hilliard equations and energy balance equation are derived within a thermodynamic framework based on entropy generation, which guarantees thermodynamic consistency. A sharp-interface limit analysis is carried out to show that the interfacial conditions of the classical sharp-interface models can be recovered from our phase-field model. Moreover, some numerical examples including thermocapillary convections in a two-layer fluid system and thermocapillary migration of a drop are computed using a continuous finite element method. The results are compared to the corresponding analytical solutions and the existing numerical results as validations for our model. The work is based on two recently published papers.

Guo, Z.L. and Lin, P. [2015], A thermodynamically consistent phase-field model for two-phase flows with thermocapillary effects, *J. Fluid Mech.*, Vol. 766, pp 226-271.

Guo, Z.L., Lin, P. and Lowengrub, J. [2014], A numerical method for the quasi-incompressible Cahn-Hilliard-Navier-Stokes equations for variable density flows with a discrete energy law, *J. Comput. Phys.*, Vol. 276, pp. 486-507.

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