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NUMERICAL INVESTIGATION OF STABILITY AND EVOLUTION OF AN EVAPORATING LIQUID FILM ON A HEATED STRUCTURED WALL

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Thin liquid films evaporating on heated structured surfaces can be found in many industrial applications. Diesel and gasoline fuels, for example, are known to form films on the cylinder surface after injection. Deposits, which have a negative effect on the combustion process, can form from those films especially in the vicinity of three-phase contact lines. Thus, it is important to understand the impact of influencing factors on film topology and stability as well as heat and mass transfer to prevent deposit formation. In this work, the effect of evaporation and wall structure on the development of liquid films and the resulting heat and mass transfer is examined. Temperature gradients develop at the liquid-gas interface and lead to Marangoni convection. Film evaporation and the unequal heating of the liquid due to the structured wall have competing effects on the film stability. Long-Wave theory is used to reduce the complexity of the problem. The resulting equation is a fourth order partial differential equation, which is solved numerically using finite differences and a Crank-Nicolson scheme. Grid and time step independence is ensured. The influence of evaporation rate and wall structure on film development and rupture time as well as heat and mass transfer within the film is discussed.

PROBLEM DEFINITION

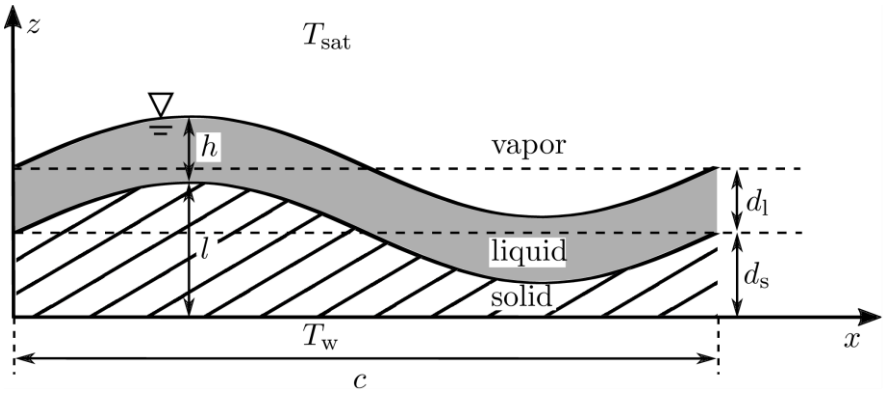

Figure 1. Sketch of the two-dimensional setup.

A sketch of the two-dimensional problem is shown in figure 1. The film is thick enough so that the van der Waals' forces do not play a role, but so thin that buoyancy can be neglected. The lower side of the wall is kept at a constant wall temperature, and the vapor phase is at the saturation temperature of the liquid. The liquid evaporates into the vapor phase so that the film height decreases with time.

The molecular-kinetic resistance at the liquid-vapor interface leads to deviations of the interface temperature from the saturation temperature and to the formation of temperature gradients. Furthermore, temperature gradients arise through uneven heating of the liquid from the structured wall. These temperature gradients give rise to Marangoni convection and may lead to film instability. The effects of evaporation and uneven heating on film development and rupture are investigated.

DERIVATION OF THE EVOLUTION EQUATION

Long-Wave theory is used to reduce the complexity of the problem, as was similarly done by Kabova et al. [2006] for non-evaporating films. Evaporating films on plane walls have been first considered in the context of long-wave theory by Burelbach et al. [1988]. In the current work, the long-wave theory is extended to cover the full system of evaporating films on heated structured walls. The governing equations for mass, momentum, and energy in fluid and wall, as well as the boundary conditions for a two-dimensional film evaporating on a heated wall, are non-dimensionalized using the following non-dimensional parameters:

$$\epsilon = \frac{d_\mathrm{l}}{c}, \qquad X = \frac{x\,\epsilon}{d_\mathrm{l}}, \qquad Z = \frac{z}{d_\mathrm{l}}, \qquad \tau = \frac{t\,\nu\,\epsilon}{d_\mathrm{l}^2}, \qquad H = \frac{h}{d_\mathrm{l}}, \qquad L = \frac{l}{d_\mathrm{l}}, \qquad Pr = \frac{\nu}{a}, \qquad \Gamma = \frac{\gamma\,k_\mathrm{l}}{\Delta h_\mathrm{v}\,d_\mathrm{l}},$$

$$G = \frac{d_\mathrm{l}^3\,g}{\nu^2}, \qquad E = \frac{k_\mathrm{l}\,(T_\mathrm{w} - T_\mathrm{sat})}{\mu\,\Delta h_\mathrm{v}}, \qquad D = \frac{3}{2}\frac{\rho_\mathrm{v}}{\rho_\mathrm{l}}, \qquad S = \frac{\sigma\,d_\mathrm{l}}{3\,\mu\,\nu}, \qquad M = -\frac{d\sigma}{dT}\frac{(T_\mathrm{w} - T_\mathrm{sat})\,d_\mathrm{l}}{2\mu\,a},$$

where ν is the kinematic and μ the dynamic viscosity, a the thermal diffusivity, σ the surface tension, k_l the thermal conductivity and ρ_l the density of the liquid. Further, Δh_v is the enthalpy of evaporation, ρ_v the density and k_v the thermal conductivity of the vapor, γ is the interfacial heat resistance divided by the enthalpy of evaporation and g the gravitational acceleration.

The governing equations and boundary conditions can be reduced to one evolution equation for the film height under the assumption that the film height is small compared to the length scale of film deformations (compare Oron et al. [1997]). This significantly reduces the complexity of the problem. The perturbation analysis for the non-dimensional film height is conducted and only terms of zeroth order are kept. The following reformulations are used:

$$F = H + L \qquad \tilde{D} = D/\epsilon^3, \qquad \tilde{E} = E/\epsilon, \qquad \tilde{G} = G\,\epsilon, \qquad \tilde{M} = M\,\epsilon, \qquad \tilde{S} = S\,\epsilon^3\,.$$

Using this, we obtain an evolution equation for the non-dimensional film height

$$H_\tau + \frac{\tilde{E}}{\Gamma + A} + \left[\left(\frac{\tilde{E}^2 A_X}{\tilde{D}(\Gamma + A)^3} + \tilde{S}(H_{XXX} + L_{XXX}) - \frac{\tilde{G}}{3}(H_X + L_X)\right) H^3\right]_X + \frac{\tilde{M}}{Pr}\left[\frac{\Gamma A_X}{(\Gamma + A)^2} H^2\right]_X = 0 \quad (1)$$

with $A = H + \dfrac{k_\mathrm{l}}{k_\mathrm{s}} L$.

The second term of the evolution equation (1) shows the influence of mass loss in the liquid due to evaporation. The third term is the effect of vapor thrust, the fourth term constitutes surface tension and the fifth term shows the influence of gravity on the film. Finally, the last term on the left hand side captures the effect of Marangoni-stresses due to the non-isothermal liquid-vapor interface.

NUMERICAL SCHEME

The evolution equation (1) is a fourth order partial differential equation in time and space. The equation is solved numerically employing a finite difference scheme using the program MATLAB. The spatial derivatives are discretised using second order central differences. The matrices containing the evaluation of the derivatives have been derived and implemented. Up to four neighbouring points are required for the calculation of the fourth derivative. A second order accurate Crank-Nicolson

method is used for time integration. This method is beneficial over implicit or explicit Eulerian methods since the Crank-Nicolson method is of second order accuracy in time. The boundary conditions on the left and right end of the domain are periodic. A grid and time step refinement study has been conducted. It can be concluded that 120 grid points in *X*-direction are sufficient to accurately capture the film shape even for highly disturbed films. Additionally, it has been found that a time step size of 0.1 is small enough to ensure independence of the results from the time integration.

RESULTS

The material and physical parameters have been chosen for a moderate evaporation rate and similar to the case considered by Kabova et al. [2006]. This leads to the following values of the non-dimensional parameters:

$$\epsilon = 0.0108, \qquad \Gamma = 1, \qquad D = 0.001, \qquad E_0 = 1.25 \cdot 10^{-5}, \qquad G = 0.01,$$
$$M = 0.25, \qquad Pr = 5, \qquad S = 1, \qquad k_l/k_s = 0\,.$$

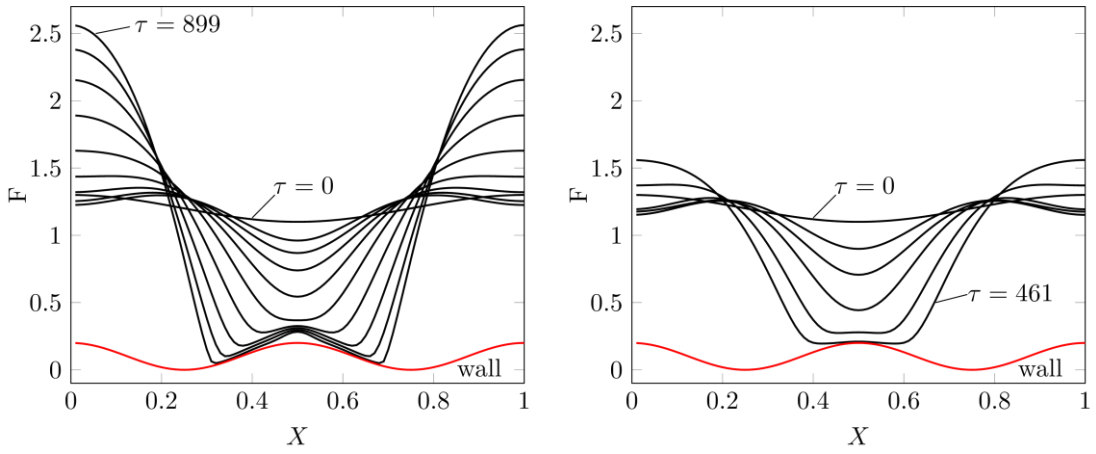

Figure 2. Time evolution for two different evaporation rates (left: $E = 0.2\,E_0$; right: $E = 1.0\,E_0$).

The development of the film height over time is shown for two different evaporation rates in Figure 2. One can see that instabilities lead to film rupture in both cases. However, the rupture time is lower for higher evaporation rates and thinning of the film from evaporation is visible. The location of the rupture point is also different in the two cases.

The nonlinear dependence of the rupture time on the evaporation rate is shown in figure 3. Film rupture occurs even if no evaporation is present. In this case rupture is caused by the Marangoni effect through the uneven heating of the film from the structured wall. The rupture time decreases rapidly with increasing evaporation rate until film thinning from evaporation governs the rupture time. Furthermore, a effect of wall structure on the rupture time can be observed. Rupture times are higher for a non-evaporating film if the wavelength of the wall structure is smaller. For higher evaporation rates, however, the rupture times are smaller when the wall structure has a smaller wave length. This behavior can be explained with interactions between the disturbance wave length and the wave-length of the wall structure as a function of film height.

The interactions between the disturbance wave length and the wall structure can be investigated further with figure 4, where the minimum film height over time is depicted for varying evaporation rates. A non-linear behavior is observed for small evaporation rates. For higher evaporation rates, the problem is governed by evaporation and the film height decreases linearly with time.

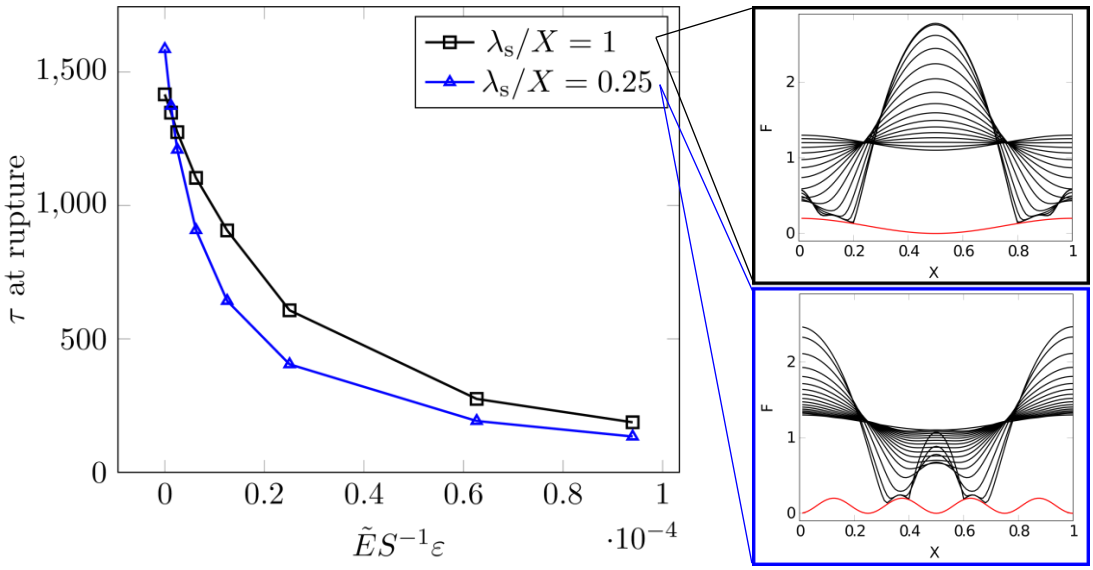

Figure 3. Rupture time over evaporation rate for different wall structures.

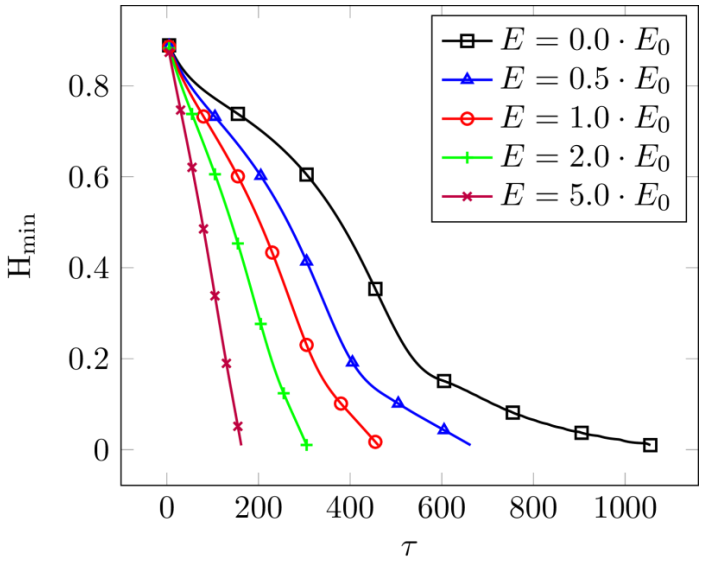

Figure 4. Minimum film height over time for varying evaporation rate.

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