COMPUTATIONAL DETERMINATION OF VOLUME AVERAGED TRANSPORT PROPERTIES OF HEAT AND FLUID FLOW IN POROUS MEDIA BY USING MICRO-TOMOGRAPHY IMAGES

Hasan CELIK*,***, Moghtada MOBEDI**, §, Akira NAKAYAMA*** and Unver OZKOL*
* Mechanical Engineering Department, Izmir Institute of Technology, Urla, Izmir, Turkey
** Faculty of Engineering, Shizuoka University, Hamamatsu, Japan
*** Mechanical Engineering Department, Shizuoka University, Hamamatsu, Japan
§Correspondence author. Fax: +81 53 478 1639 Email: moghtada.mobedi@shizuoka.ac.jp

ABSTRACT In this study, the theory and techniques for obtaining VAM (Volume Average Theory) transport properties of a porous medium from micro-tomography images are described. The validation of the results with reported experimental or numerical values in literature may not be sufficient, hence a comprehensive attention is paid to the techniques that can be used for verification of the obtained numerical results at each step of this long computational process. The suggested verification techniques are categorized and explained in details.

BACKGROUND

Heat and fluid flow in porous media has wide applications from nature to industry. The flow of air in a human lung, flow of water in soil, flow of hot air through fixed packed beds for drying purpose or even airflow between tree trunks in forests are some examples of heat and fluid flow in porous environment. There are two main theoretical approaches to analyse heat and fluid flow in a porous structure known as Pore Scale Method (PSM) and Volume Averaged Method (VAM) as shown in Figure 1. In PSM, Figure 1(a), the conservation of mass and momentum equations are solved for the fluid flowing through the voids of the porous medium while the energy equation is solved for both solid and fluid. The PSM yields accurate distributions for velocity, pressure and temperature. However, its practical application for porous media involving numerous pores and voids is not feasible for the time being.

Figure 1. Two main theoretical approaches to analyse heat and fluid flow in a porous structure a) Pore Scale Method (PSM), b) Volume Averaged Method (VAM).
This difficulty of PSM leads researches to develop and employ VAM [Mobedi et al. 2016]. Volume Average Method for porous media (see Figure 1(b)) has been developed based on Volume Average Theory. The results of VAM may not provide an exact view for temperature, pressure and velocity in the porous media since the space averaged values are used. However, the method is practical and the accuracy of results is generally in acceptable ranges. The VAM equations for a heat and fluid flow field can be expressed as:

$$\nabla \cdot \langle \vec{u} \rangle = 0$$  \hspace{2cm} (1)

$$\frac{i}{\varepsilon} \frac{\partial \langle \vec{u} \rangle}{\partial t} + \frac{1}{\varepsilon^2} \langle \vec{u} \rangle \cdot \nabla \langle \vec{u} \rangle = - \frac{1}{\rho_f} \nabla \langle p \rangle + \frac{\mu}{\varepsilon} \nabla^2 \langle \vec{u} \rangle - \frac{\mu}{\rho_f} \frac{K}{K^{1/2}} \langle \vec{u} \rangle \langle \vec{u} \rangle$$  \hspace{2cm} (2)

$$\rho_f c_p f \left( \varepsilon \frac{\partial \langle T \rangle^f}{\partial t} + \langle \vec{u} \rangle \cdot \nabla \langle T \rangle^f \right) = (k_{stag,f} + k_{dis}) \nabla^2 \langle T \rangle^f + h_{int} \epsilon_{int} \left( \langle T \rangle^s - \langle T \rangle^f \right)$$  \hspace{2cm} (3)

$$\rho_s c_p s (1 - \varepsilon) \frac{\partial \langle T \rangle^s}{\partial t} = k_{stag,s} \nabla^2 \langle T \rangle^s - h_{int} \epsilon_{int} \left( \langle T \rangle^s - \langle T \rangle^f \right)$$  \hspace{2cm} (4)

As can be seen from the above equations, for application of VAM, the transport parameters (permeability, Forchheimer coefficient, interfacial heat transfer coefficient and effective thermal conductivity) which can be called as VAM transport parameters should be known. The use of pore scale simulations of heat and fluid flow for the determination of VAM transport parameters has become popular in recent years due to developments in both computer technology and computational methods [Mobedi et al. 2016].

**CONSIDERED DOMAIN AND COMPUTATIONAL DETAILS**

In this study, the micro-tomography images of two metal foams with 10 and 20 PPI are obtained and their digital representations are created in computer environment. Firstly, sample of metal foam of 20 PPI is recruited as shown in Figure 2(a). Once, the tomography images are obtained for the considered domain with size of 12 x 12 x 12 mm$^3$, 3D representation of the metal foam can be built, as shown in Figure 2(b). Then, mesh can be built for the solid and fluid domains of the created 3D metal foam and it can be imported to a CFD solver.

![Figure 2](image.png)

Figure 2. A metal foam and digital representation, a) the metal foam structure with 20 PPI, b) the digital representation by micro-tomography technique, c) generated mesh for fluid region.

The numerical model is shown in Figure 3. The governing equations for the imported domain are solved by a commercial CFD solver and consequently the temperature, velocity and pressure distributions for entire domain are obtained. The computational domain is divided to sufficient number of sub-volumes and the volume average values of dependent parameters for sub-volumes are obtained. Then, VAM
transport parameters are calculated according to their mathematical definitions. There is a good agreement between the calculated VAM transport properties and reported experimental and theoretical results in literature.

![Graphical representation of computational domain and boundary conditions](image)

**Figure 3.** The considered computational domain and boundary conditions.

**DISCUSSION**

Figure 4 shows contours of velocity, pressure and temperature of the computational domain. As it can be seen in Figure 4(a), the velocity is uniform at inlet section, the homogeneity of velocity is destroyed through the metal foam region due to the change of fluid flow area. The change of pressure inside the computational domain can be seen in Figure 4(b). The pressure at the inlet region is high and it decreases through the flow direction. The pressure considerably drops in the porous region and its gradient is negligible at inlet and outlet regions. The temperature contours are shown in Figure 4(c). At inlet section, the temperature is kept constant, the fluid is heated in metal foam due to the higher surface temperature of the metal foam. The heated fluid flows through the outlet of the channel and the effect of convection at the outlet region can also be seen.

![Sample of obtained contours](image)

**Figure 4.** A sample of obtained contours a) velocity, b) pressure, c) temperature.

Since the numerical procedure by using micro-tomography image is complicated, it is prone to errors, and therefore, it requires some checking methods to verify the correctness of the results. In this study, the practical checking methods at each step of computational process are suggested. For instance, the velocity and fluid flow area changes in a cross-section (which is perpendicular to flow direction) can be plotted as seen from Figure 5(a). By decreasing of fluid flow area, the fluid velocity should increase particularly for isotropic metal foams. Additionally, the macroscopic pressure drop through the domain
should be calculated and plotted for entire domain (Figure 5(b)). The main pressure drop should occur in metal foam and its change should be linear showing existence of a fully develop flow. No doubt that, the application of linear curve fitting to the large number of calculated averaged (macroscopic) pressure in flow direction in porous media yields accurate gradient of pressure. This macroscopic pressure drop at two or three different locations can be used and the values of permeability and inertia coefficient can be calculated. Moreover, in order to judge the results objectively, the plotting of dimensionless pressure drop with Re number is strongly recommended (see Figure 5(c)). This diagram facilitates a) discovering of any error in computational process, b) distinguishing of Darcy and non-Darcy regions, and c) obtaining accurate value from applied curve fitting. As it can be seen, several methods not only for hydraulic but also for thermal transport parameters can be applied to eliminate any possible error.

![Figure 5. Some checking techniques should be performed for calculation macroscopic transport parameters, a) opposite behaviour of macroscopic velocity and free cross section area, b) check of pressure drop and existence of fully develop, c) check of Darcy and non-Darcy regions.](image)

CONCLUSION

The volume averaged transport properties of heat and fluid flow for a metal foam are calculated successfully by using micro-tomography image. It is observed that the risk of both computational and conceptual errors is high and definitely some checks should be performed at each step of process. In addition to the classical validations such as grid independency, some checks such as the change of velocity with variation in fluid flow area, existence of hydraulically and thermally fully developed flow, indicating number of voxels in pore, the effect of inertia on velocity profiles in pore, observation of Darcy-non Darcy regions should be performed. All these checking methods and techniques with their benefits are described in this study.

ACKNOWLEDGEMENT

Hasan Celik would like to thank to the generous support of Scientific and Technological Research Council of Turkey (TUBITAK) under 2214/A fellowship. Furthermore, the supercomputer system of Academic Center for Computing and Media Studies of Kyoto University in Japan and TUBITAK ULAKBIM, High Performance and Grid Computing Center (TRUBA resources) in Turkey are used. Authors would like to express their gratitude to these organizations.

REFERENCES