

## A COMBINED ANALYTICAL-NUMERICAL MODEL TO DETERMINE RADIATIVE PROPERTIES OF TETRAKAIDECAHEDRA CELL STRUCTURE

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**ABSTRACT** In the present work, radiative properties of representative porous media are estimated using pore structure modelling along with radiation interaction with the solid matrix. This approach is found to be more useful as it enables to understand the insight physics of the radiation propagation. The tetrakaidecahedra unit cell structure is found to be a close approximation of actual foam structure and hence in the present work, it is considered as a representative unit cell structure. An analytical model based on zonal method is developed to determine effective reflectance ( $\rho_{e,c}$ ) and transmittance ( $\tau_{e,c}$ ) of the considered unit cell structure. The results of unit cell are then extended to multi-layered unit cell structures using recurrence relationships. The radiative properties of an equivalent homogeneous participating medium are determined using inverse method. The radiative transfer equation (RTE) in a homogeneous participating medium is solved using the finite volume method (FVM). The genetic algorithm (GA) required in the inverse method is integrated to the FVM code. The obtained results of recurrence relationships are used in the FVM-GA code to determine extinction coefficient ( $\beta^*$ ) and scattering albedo ( $\omega^*$ ) of an equivalent homogeneous participating medium. Effects of solid reflectivity ( $\rho_s$ ) and pore density on the radiative properties of the porous media have been studied using the proposed model. Results obtained from this study show that the  $\beta^*$  decreases with  $\rho_s$  while  $\omega^*$  shows increasing trend with  $\rho_s$ . With increase in pore density,  $\beta^*$  increases. However,  $\omega^*$  is found to be insensitive to pore density values.

### NOMENCLATURE

a, b, c, d, e	coefficients defined in Eqs. 9-11	
b	source term	
$\hat{e}$	unit vector	
q	radiation heat flux	(W/m <sup>2</sup> )
$\vec{r}$	position vector	(m)
$\hat{s}$	direction vector for radiation intensity	
S	source function	(W/m <sup>3</sup> )
$\Delta V$	volume of a control volume	(m <sup>3</sup> )

### GREEK SYMBOLS

$\varepsilon$	porosity, emissivity	
$\theta$	polar angle	(rad)
$\rho$	reflectance	

$\rho_s$	solid reflectivity	
$\tau$	transmittance	
$\phi$	azimuthal angle	(rad)

### SUBSCRIPTS

b	blackbody, reflected back, bottom cell face
B	neighbouring nodes of CV in bottom direction
HPM	homogeneous participating media
i	incident
n	number of layers
P	central node of CV
t	transmitted, top cell face
T	neighbouring nodes of CV in top direction

### ABBREVIATIONS

CV	control volume
D	dimension
GA	genetic algorithm
FVM	finite volume method
PPC	pores per centimeter
RTE	radiative transfer equation

### INTRODUCTION

The characteristics of an open cell foam such as high surface area to volume ratio and high tortuosity enhance its heat transfer and flow mixing capabilities. Hence, open cell foams can be envisioned as convection to radiation convertors (C-R-C) in compact heat exchangers [Zhang et al. 1997], and volumetric solar receiver [Fend et al. 2004]. Other high temperature applications of the open cell foam can be mentioned in porous radiant burners [Gauthier et al. 2008], fire barriers [Lu and Chen 1999, Coquard et al. 2012], and porous media combustion [Brenner et al. 2000]. Radiation is found to be dominant mode of heat transfer in such high temperature applications. Efficiency and effectiveness of such systems can be improved if radiative properties of the porous media such as extinction coefficient ( $\beta$ ), scattering albedo ( $\omega$ ) and scattering phase function ( $\Phi$ ) are known beforehand. The radiative properties of the porous media are affected by the various parameters such as porosity, pore density, pore-structure and optical properties of the solid matrix. Hence, it is desirable to have some microscopic model which can explain radiative behaviour of the porous media under the influence of change in any of these affecting parameters. Performance of such models mainly depends on accurate modelling of porous structure as well as accuracy of the method considered to study radiative interactions with the solid matrix.

Zhao et al. [2008] developed an explicit analytical model to determine radiative conductivity of an open cell foam represented by cubic cell structure. Accuracy of the Zhao's model was improved by Contento et al. [2014a]. They introduced Monte-Carlo ray tracing method to calculate radiation view factors. Moreover, both view factors and coefficients of the model were calculated iteratively. This model is later extended to tetrakaidehedra structure [Contento et al. 2014b] which is considered to be more accurate approximation of actual open cell foam structure. The models developed by Zhao et al. [2008] and Contento et al. [2014a, 2014b] did not involve any equivalent homogeneous medium approach. Instead, it is based on purely geometrical considerations.

The hypothesis of an equivalent homogeneous media approach in conjunction with analytical methods are used by various researchers. Fu et al. [1997] developed a combined analytical-numerical model to estimate radiative properties of structured porous media. The considered representative unit cell was made up of a cuboid voided by a centrally placed sphere. The radiative heat transfer in the unit cell was solved using zonal method. The obtained results of effective reflectance and transmittance were used with discrete ordinates method to determine radiative properties of an equivalent homogeneous participating medium (HPM). Inspired from the work of Fu et al. [1997], Patel and Talukdar [2016] developed a numerical model to determine radiative properties of a cubic cell porous media. They introduced a finite volume method (FVM) integrated with the blocked-off region approach to solve radiative heat transfer in the considered porous structure. The radiative properties of homogeneous medium were obtained using inverse method (genetic algorithm) integrated to finite volume method of RTE.

In the present work, a combined analytical-numerical model developed by Fu et al. [1997] is extended to the tetrakaidehedra structure. The zonal method based an analytical model is developed to calculate effective reflectance ( $\rho_{e,c}$ ) and transmittance ( $\tau_{e,c}$ ) of the tetrakaidehedra unit cell structure. Using recurrence relationships, the results of  $\rho_{e,c}$  and  $\tau_{e,c}$  are extended to porous media of different pore densities. In the present work, pore density is defined in terms of pores per centimeter (PPC) which varies from 2 to 8. The radiative properties of the porous media are determined in such a way that the considered porous media can be assumed as a homogeneous participating medium. The requirement of scattering phase function  $\Phi$  in the radiative heat transfer equation (RTE) can be eliminated if the directional dependence of scattering phenomena is inherently taken care by the values of  $\beta$  and  $\omega$ . Such  $\beta$  and  $\omega$  are called weighted extinction coefficient ( $\beta^*$ ) and weighted scattering albedo ( $\omega^*$ ). The 1-D FVM code developed to solve RTE in a participating medium is coupled with the genetic algorithm (GA). The GA adjusts the values of  $\beta^*$  and  $\omega^*$  such that the reflectance and transmittance of HPM match well with the results of porous media of considered pore density. The effect of solid reflectivity ( $\rho_s$ ) and pore density (PPC) on radiative properties of the porous media are investigated. The obtained results of  $\beta^*$  and  $\omega^*$  are compared with the results of cubic cell porous structure.

Determination of radiative properties of porous media using microscopic approach is preferred over macroscopic approach because it enables to understand the insight physics of the radiation propagation through the complex pore structure. The accuracy of the microscopic approach mainly depends on (i) accuracy in structural modelling and (ii) accuracy of the method used to investigate radiative interactions with the solid matrix. The complex structure of the open cell porous media can be approximated into idealised unit cell structure. Some of the widely used idealized porous structures are cubic cell structure, Dul'nev's unit cell, dodecahedra cell, tetrakaidehedra and Weaire-Phelan cell structure. Among these tetrakaidehedra and Weaire-Phelan cell structure are proved to be close approximation of actual open cell foam structure. In present work, the tetrakaidehedra unit cell structure, shown in Fig. 1, is considered as a representative foam structure. The porous media are usually characterized using two important physical parameters (i) porosity and (ii) pore density. The porosity represents the total void fraction while the pore density represents the number of pore cells accommodate in a unit length. Generally the pore density is represented in terms of PPI (pores per inch) or PPC (pores per centimeter). In present work, considered unit cell has porosity of around 0.98. The effect of pore density on radiative properties of the porous media are studied by considering different values of PPC ranging from 2 to 8. Moreover, the dependency of radiative properties on solid reflectivity ( $\rho_s$ ) is investigated by calculating  $\beta^*$  and  $\omega^*$  against different values of  $\rho_s$  varying from 0.5 to 0.99.

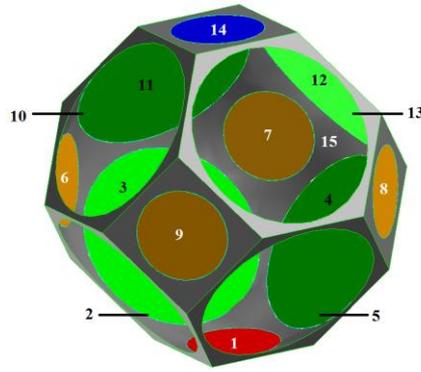


Figure 1. The tetrakaidecahedra unit cell structure

## METHODOLOGY

The radiative interaction in the porous structure can be studied by analytical or by numerical methods. Fu et al. [1997] used zonal method to study radiative heat transfer in a cubic unit cell structure. The cubic cell considered by Fu et al. [1997] can be generated by subtracting solid sphere from the solid cuboid of unit size. In present study, similar model is extended for more accurate foam structure i.e., the tetrakaidecahedra structure. Along with cold solid matrix consideration, other assumptions made by Fu et al. [1997] can be restated for present work are (i) The wavelength of the radiation is assumed to be much smaller than the dimension of unit cell, (ii) the reflection from the solid matrix is assumed to be diffuse, (iii) the radiation properties are independent of temperature conditions, (iv) infinite number of unit cells are arranged in planar direction and hence, the model can be considered as a one-dimensional, and (v) scattering of radiation is considered to be isotropic.

The proposed model can be divided into three crucial steps. (i) Determination of reflectance and transmittance using zonal method, (ii) determination of effective reflectance and transmittance of 'n' layers of unit cell structure using recurrence relationships and (iii) identification of radiative properties of an equivalent homogeneous participating media using genetic algorithm (GA) integrated with FVM of the standard RTE. The detailed descriptions of each step are as follows.

**(i) Determination of Effective Reflectance and Transmittance of a Tetrakaidecahedra Unit Cell Structure:** The tetrakaidecahedra unit cell structure shown in Fig. 1 consists of solid matrix along with 14 windows. The bottom and the top windows are numbered as '1' and '14' and serves as target windows to calculate reflectance and transmittance of the unit cell structure. The windows on the bottom inclined hexagons are numbered as 2, 3, 4, and 5. Likewise windows of the top inclined hexagons are identified as 10, 11, 12 and 13. The remaining windows of the transverse vertical face of the unit cell structure are numbered as 6, 7, 8 and 9. The solid surface is commonly numbered as surface '15'. The values of effective reflectance and transmittance of a tetrakaidecahedra unit cell structure are calculated using the zonal method. From the bottom window '1', radiation heat flux of a unit magnitude ( $q_i = 1 \text{ W/m}^2$ ) is entered into the unit cell. The radiation heat flux intercepted by other windows '2-14' and solid surface '15' can be calculated by determining all possible configuration factors. Depending on the value of solid reflectivity ( $\rho_s$ ), part of the falling radiation is reflected back by the surface '15' while the remainder is absorbed by the solid matrix. In identification approach, the solid matrix is assumed to be at cold condition and as a results, emission of radiation from the solid matrix can be neglected.

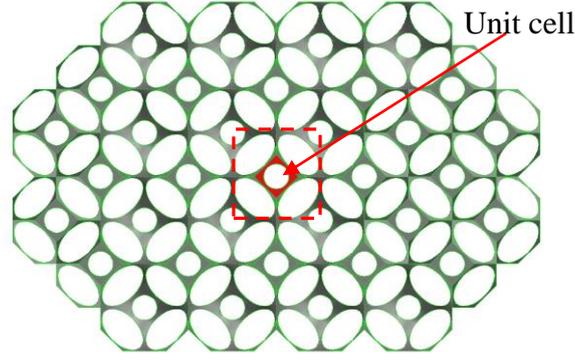


Figure 2. Top view of the unit cell arrangements.

The assumption of large number of unit cells on the planar directions (Fig. 2) allow us to implement symmetry boundary conditions on the planar windows '2-13'. In other words, the amount of radiation approaches to planar windows '2-13' are reflected back to the unit cell without any attenuation. The reflection from the solid matrix as well as from the transverse windows are assumed to be diffuse. The radiosity of any surface 'i' can be expressed as algebraic sum of the emission and reflection.

$$J_i = \varepsilon_i E_{b,i} + \rho_i G_i \quad (1.a)$$

Since, the solid matrix is assumed to be cold, the emission term in the above equation can be neglected. Moreover, the irradiation term  $G_i$  can be expressed in terms of radiosity of all other windows and surface '15'. Hence, the radiosity term  $J_i$  can be written as:

$$J_i = \rho_i \sum_{j=1}^n J_j F_{ij} \quad \begin{cases} \rho_i = 1, & i = 2 - 13 \\ \rho_i = \rho_s, & i = 15 \end{cases} \quad (1.b)$$

where,  $F_{ij}$  is the configuration factor of surface/window  $i$  to surface/window  $j$ .  $\rho_i$  is reflectivity of the surface 'i'. Due to symmetry boundary conditions on the planar windows '2-13', the radiosity of these windows are equal to the irradiation  $G_i$ . Moreover, taking the advantage of symmetry of the unit cell structure, following simplifications can be made:

$$J_2 = J_3 = J_4 = J_5 = J_{10} = J_{11} = J_{12} = J_{13} \quad (2.a)$$

$$J_6 = J_7 = J_8 = J_9 \quad (2.b)$$

After the energy balance, the radiosity of window 2, 6 and surface 15 can be expressed as:

$$J_6 = F_{1-6} J_1 + (2F_{1-6} + F_{1-14}) J_6 + 8F_{1-2} J_2 + F_{1-15} J_{15} \quad (3)$$

$$J_2 = F_{2-1} J_1 + 4F_{2-1} J_6 + 7F_{2-3} J_2 + F_{2-15} J_{15} \quad (4)$$

$$J_{15} = \rho_s (F_{15-1} J_1 + 4F_{15-1} J_6 + 8F_{15-2} J_2 + F_{15-15} J_{15}) \quad (5)$$

where,  $J_1 = q_1 = 1 \text{ W/m}^2$ .

The simultaneous solution of Eqs. 3 to 5 are:

$$J_6 = \frac{(a.b+8c.d)J_1}{(e.b-32c.d)} \quad (6)$$

$$J_2 = \frac{dJ_1+4.dJ_6}{b} \quad (7)$$

$$J_{15} = \frac{\rho_s(F_{15-1} J_1 + 4F_{15-1} J_6 + 8F_{15-2} J_2)}{(1 - \rho_s F_{15-15})} \quad (8)$$

where, coefficients a, b, c, d and e in above equations are:

$$a = (F_{1-6} \cdot (1 - \rho_s F_{15-15})) + (\rho_s F_{1-15} F_{15-1}) \quad (9)$$

$$b = ((1 - 7F_{2-3}) \cdot (1 - \rho_s F_{15-15})) - (8\rho_s F_{15-2} F_{2-15}) \quad (10)$$

$$c = (F_{1-2} \cdot (1 - \rho_s F_{15-15})) + (\rho_s F_{1-15} F_{15-2}) \quad (11)$$

$$d = (F_{2-1} \cdot (1 - \rho_s F_{15-15})) + (\rho_s F_{15-1} F_{2-15}) \quad (12)$$

$$e = ((1 - 2F_{1-6} - F_{1-14}) \cdot (1 - \rho_s F_{15-15})) - (4\rho_s F_{1-15} F_{15-1}) \quad (13)$$

After the multiple reflections from the surface '15' and windows '2-13', part of the incoming radiation ( $q_i$ ) radiation is transmitted to the top window '14' ( $= q_t$ ). Some fraction of the  $q_i$  is reflected back to the bottom window '1' ( $= q_b$ ), while the remainder is absorbed by the solid matrix. The quantities  $q_b$  and  $q_t$  can be calculated using following equations:

$$q_1 = q_b = 4F_{1-6} J_6 + 8F_{1-2} J_2 + F_{1-15} J_{15} \quad (14)$$

$$q_{14} = q_t = F_{1-6} J_1 + 4F_{1-6} J_6 + 8F_{1-2} J_2 + F_{1-15} J_{15} \quad (15)$$

The values of  $q_r$  and  $q_t$  calculated using above equations are used to determine effective reflectance ( $\rho_{e,c}$ ) and transmittance ( $\tau_{e,c}$ ) of the unit cell structure.

$$\rho_{e,c} = \frac{q_b}{q_i} \quad \text{and} \quad \tau_{e,c} = \frac{q_t}{q_i} \quad (16)$$

**(ii) Determination of Effective Reflectance and Transmittance of 'n' Layers of Unit Cell Structures:** The effect of pore density on radiative properties of the porous media can be well understood by extending the results of unit cell to the 'n' layers of porous structure. The pore density can be expressed in terms of pores per inch (PPI) or pores per centimeter (PPC). In present work, pore density is defined in terms of PPC which varies from 2 to 8. The recurrence relationships [Patel and Talukdar 2016] can be used to obtain effective reflectance ( $\rho_n$ ) and transmittance ( $\tau_n$ ) of the desired pore density. Here, subscript 'n' represents the number of unit cells calculated from the considered pore density.

$$\rho_{e,n} = \rho_{e,n-1} + \frac{\rho_{e,c} \tau_{e,n-1}^2}{1 - \rho_c \rho_{e,n-1}} \quad (17a)$$

$$\tau_{e,n} = \frac{\tau_{e,c} \tau_{e,n-1}}{1 - \rho_c \rho_{e,n-1}} \quad (17b)$$

where,  $\rho$  and  $\tau$  with subscript 'n-1' stands for reflectance and transmittance values of 'n-1' layers of unit cell structures.  $\rho_{e,c}$  and  $\tau_{e,c}$  are reflectance and transmittance of the unit cell calculated using Eqs. (16).

**(iii) Determination of Radiative Properties of an Equivalent Homogeneous Participating Media:** If radiative properties ( $\beta^*$  and  $\omega^*$ ) of a hypothetical homogeneous participating medium (HPM) are such that the values of reflectance ( $\rho_{HPM}$ ) and transmittance ( $\tau_{HPM}$ ) obtained for HPM match well with the values of  $\rho_n$  and  $\tau_n$  (obtained using Eqs. 17(a) and (b)), then the medium can be considered as an equivalent HPM. The main direction of radiation propagation is from the bottom window to the top. Hence, an equivalent HPM can be considered as one-dimensional. The 1-D HPM is assumed to be

bounded between bottom (hot) and top (cold) boundaries. From the bottom boundary, radiation heat flux of unit magnitude ( $q_i = 1 \text{ W/m}^2$ ) is entered into the participating media. The finite volume method based numerical method is used to solve the radiative heat transfer equation (RTE) in the participating medium. The RTE can be written as:

$$\frac{dI(\vec{r}, \hat{s})}{ds} = -\beta(\vec{r}) I(\vec{r}, \hat{s}) + S(\vec{r}, \hat{s}) \quad (18)$$

where,  $\beta$  is extinction coefficient representing the combined effect of absorption and out-scattering ( $\beta(\vec{r}) = \kappa(\vec{r}) + \sigma(\vec{r})$ ). The source terms  $S(\vec{r}, \hat{s})$  in the above equation can be defined as:

$$S(\vec{r}, \hat{s}) = \kappa(\vec{r}) I_b(\vec{r}) + \frac{\sigma(\vec{r})}{4\pi} \int I(\vec{r}, \hat{s}') \Phi(\hat{s}', \hat{s}) d\Omega' \quad (19)$$

The terms  $\vec{r}$  and  $\hat{s}$  are position and unit vectors of the directional radiative intensity.  $I_b(\vec{r})$  is the black body radiation emission term. In present method, solid matrix of the unit cell is assumed to be cold. Hence, the black body radiation term can be neglected.  $\kappa(\vec{r})$  and  $\sigma(\vec{r})$  are called absorption and scattering coefficients. While,  $\Phi$  is called scattering phase function. In present work, scattering is considered to be isotropic. As a result, the corresponding scattering phase function ( $\Phi$ ) can be taken as unity. The intensity direction  $\hat{s}$  is defined as:

$$\hat{s} = (\sin\theta\cos\phi)\hat{e}_x + (\sin\theta\sin\phi)\hat{e}_y + (\cos\theta)\hat{e}_z \quad (20)$$

where,  $\theta$  and  $\phi$  are the zenith and azimuthal angle respectively. The FVM developed by Chai and Patankar [2000] is used to solve the standard RTE in homogeneous participating media. The discretized form of RTE for one dimensional heat transfer (along z direction) can be written as:

$$a_p^l I_p^l = a_B^l I_B^l + a_T^l I_T^l + b^l \quad (21)$$

The neighbouring coefficients and node coefficient can be expressed as:

$$\begin{aligned} a_T^l &= \max(0, -A_t D_{ct}^l, 0), \quad a_B^l = \max(0, -A_b D_{cb}^l, 0) \\ a_p^l &= \max(A_t D_{ct}^l, 0) + \max(A_b D_{cb}^l, 0) + \beta_p \Delta V_p \Delta \Omega^l \end{aligned} \quad (22)$$

The source term  $b^l$  in Eq. (21) can be written as:

$$b^l = S_p^l \Delta V_p \Delta \Omega^l \quad (23)$$

The discretised form of  $S_p^l$  in the absence of black body radiation can be written as:

$$S_p^l = \frac{\sigma_p}{4\pi} \sum_{i=1}^l I_p^{l'} \Delta \Omega^l \quad (24)$$

The direction cosine functions are expressed as:

$$D_{ct}^l = \int_{\Delta \Omega^l} (\hat{s}^l \cdot \hat{e}_t) d\Omega \quad \text{and} \quad D_{cb}^l = -D_{ct}^l \quad (25)$$

More details of the discretization procedure of RTE can be found in the work of Talukdar et al. [2005]. The convergence criteria is defined in terms of maximum difference of radiation intensity between two consecutive iterations. If the difference reduces to  $1e-6$ , the solution is considered to be converged. Once the solution is converged, the amount of radiation heat flux transmitted to the top boundary and the amount of radiation heat flux reflected back to the bottom boundary can be calculated using the following equations:

$$q_t = \int_{\vec{r} \cdot \hat{e}_z > 0} I(\vec{r}, \hat{s})(\vec{r} \cdot \hat{e}_z) d\Omega \quad \text{and} \quad q_b = \int_{\vec{r} \cdot \hat{e}_z < 0} I(\vec{r}, \hat{s})(\vec{r} \cdot \hat{e}_z) d\Omega \quad (26)$$

The values of  $q_b$  and  $q_t$  obtained using the above equations can be used to determine effective reflectance ( $\rho_{\text{HPM}}$ ) and transmittance ( $\tau_{\text{HPM}}$ ) of the homogeneous participating media.

$$\rho_{\text{HPM}} = \frac{q_b}{q_i} \quad \text{and} \quad \tau_{\text{HPM}} = \frac{q_t}{q_i} \quad (27)$$

The solution of RTE in the HPM starts with the random values of absorption ( $\kappa$ ) and scattering coefficient ( $\sigma_s$ ). The genetic algorithm (GA) incorporated to the FVM code of RTE adjusts the values of  $\kappa$  and  $\sigma_s$  such that the objective function ( $J$ ) defined in Eq. (28) can be minimized.

$$J = (\rho_{e,n} - \rho_{\text{HPM}})^2 + (\tau_{e,n} - \tau_{\text{HPM}})^2 \quad (28)$$

The developed GA has population size of 10 and stopping criteria of  $1e-9$ . The flow chart shown in Fig. 3 summarizes all the steps used in the proposed model.

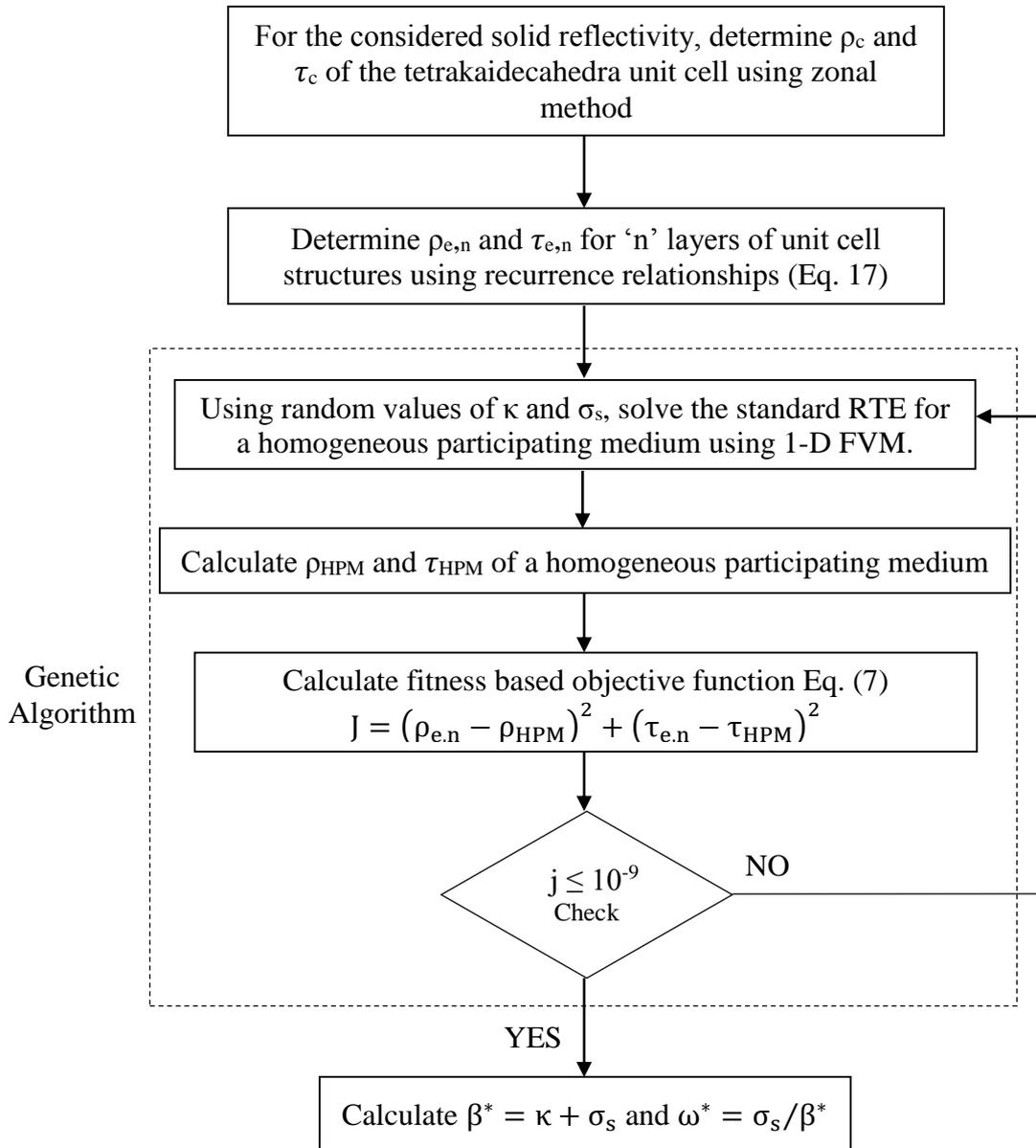


Figure 3. Flow chart to estimate radiative properties of the tetrakaidecahedra structure

## RESULTS AND DISCUSSION

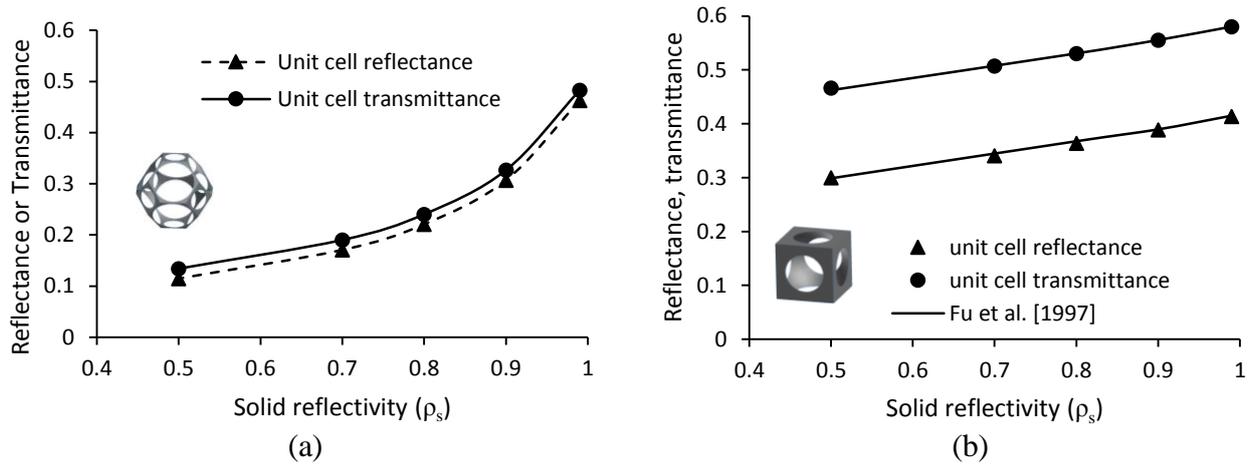


Figure 4. Effective reflectance and transmittance of (a) tetrakaidecahedra ( $\epsilon = 0.98$ ) and (b) cubic unit cell structure ( $\epsilon = 0.95$ ) as a function of solid reflectivity ( $\rho_s$ )

Figure 4 (a) shows variation of effective reflectance ( $\rho_{e,c}$ ) and transmittance ( $\tau_{e,c}$ ) of a tetrakaidecahedra unit cell with solid reflectivity ( $\rho_s$ ). As  $\rho_s$  increases, both reflectance and transmittance show increasing trend. The increasing trend of  $\rho_{e,c}$  and  $\tau_{e,c}$  can be explained by the fact that for low  $\rho_s$  values, the amount of radiation absorbed by the solid matrix is high. Consequently, the amount of radiation reflected from the solid surface ‘15’ is less. With increasing  $\rho_s$ , the amount of radiation reflected from the solid surface ‘15’ increases. As a result, the amount of radiation heat flux reaches to the top and the bottom window increases. It is also observed that both reflectance and transmittance have identical trend, and at any  $\rho_s$ , the  $\tau_{e,c}$  is slightly higher than the  $\rho_{e,c}$ . The observed trend can be supported by the fact that the bottom and the top windows share equal amount of configuration factors with the other windows (2-13) and surface ‘15’. As a result, the reflected radiation from the windows ‘2-13’ and solid surface ‘15’ are equally distributed to both top and bottom windows. However, some fraction of the incoming  $q_i$  from the bottom window ‘1’ is directly transmitted to the top window ‘14’. As a result, the  $\tau_{e,c}$  is slightly higher than  $\rho_{e,c}$ . The difference in the values of  $\tau_{e,c}$  and  $\rho_{e,c}$  mainly depends on the configuration factor  $F_{1-14}$ . Similar trend was observed in the work of Fu et al. [1997] for a cubic unit cell structure. Porosity of the cubic unit cell was varied by varying radius of the subtracting sphere. The maximum porosity that can be achieved for the cubic unit cell is 0.95. The combined analytical-numerical model developed by Fu et al. [1997] is repeated here for the cubic unit cell ( $\epsilon = 0.95$ ) to gain confidence in the proposed model. The obtained results of effective reflectance and transmittance agree very well with the published results as shown in Fig. 4(b). It is important to note that in spite of having higher porosity, the tetrakaidecahedra structure offers low values of unit cell reflectance and transmittance compared to the cubic unit cells structure, especially at low  $\rho_s$  values. The main reason behind this contradiction can be explained by observing surface area to strut volume ratios (SA:V) of both unit cell structures. The SA:V of the tetrakaidecahedra structure is very high ( $\sim 88$ ) compared to that of the cubic unit cell structure ( $\sim 16$ ). This shows that the solid matrix of tetrakaidecahedra structure is uniformly distributed within the unit cell volume and actively participate in radiative interaction. While in case of the cubic unit cell structure, large portion of the solid volume is remain hidden for radiation interaction.

The results of reflectance ( $\rho_c$ ) and transmittance ( $\tau_c$ ) of the tetrakaidecahedra unit cell structure are extended for ‘n’ layers of unit cell structures using recurrence relationships shown in Eqs 17(a) and 17(b). The values of ‘n’ in the recurrence relationships depends on the values of desired pore density. In present work, pore densities are varied from 2 to 8. The obtained values of  $\rho_{e,n}$  and  $\tau_{e,n}$  are used in the inverse method to determine extinction coefficient  $\beta^*$  and scattering albedo  $\omega^*$  of an equivalent homogeneous participating medium.

The standard RTE are solved using FVM proposed by Chai and Patankar [2000]. The one dimensional domain of homogenous participating media has dimension of 1 centimeter. The domain is discretized into 1500 control volumes. The angular space are discretized by considering 16 rays in the azimuthal angle ( $0^\circ$  to  $360^\circ$ ) and 20 rays in the zenith angle ( $0^\circ$  to  $180^\circ$ ). The selection of large number of CVs and ray angles ensure that the results are free from grid effect and ray effects.

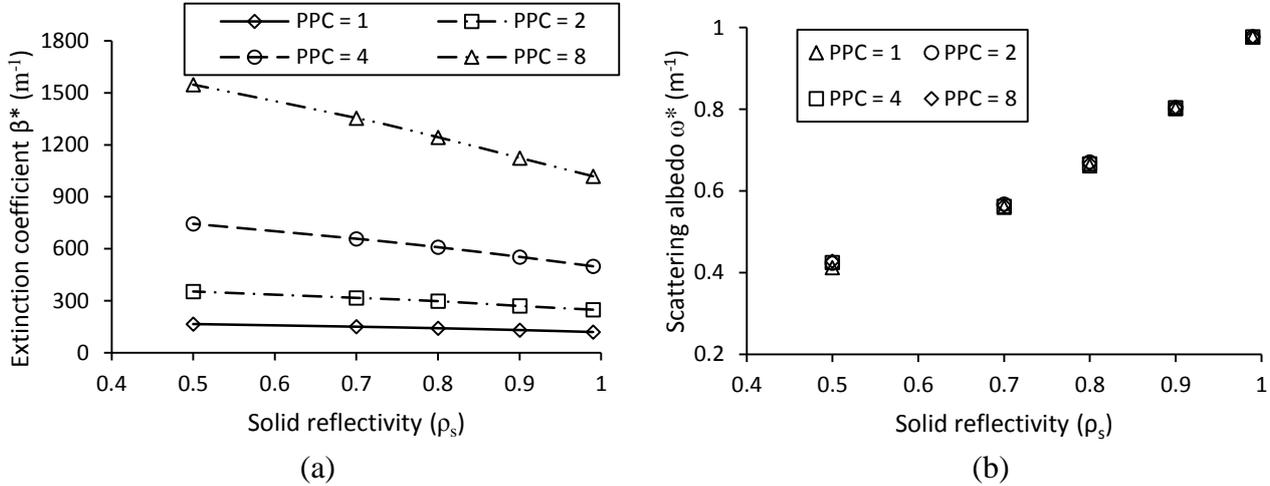


Figure 5. Variations of (a) extinction coefficient ( $\beta^*$ ) and (b) scattering albedo ( $\omega^*$ ) as a function of solid reflectivity ( $\rho_s$ ) at different PPC for a tetrakaidehedra structure

Figure 5 (a) shows variation of extinction coefficient ( $\beta^*$ ) as a function of solid reflectivity  $\rho_s$  at different pore density values. The extinction coefficient shows decreasing trend with increase in  $\rho_s$  values. With increase of  $\rho_s$ , the amount of radiation reflected by the solid surface increases. Hence, corresponding scattering coefficient ( $\sigma_s$ ) of an equivalent participating media increases. However, the increase of scattering coefficient ( $\sigma_s$ ) is at the expense of decrease of absorption coefficient ( $\kappa$ ). Since, the decrease in  $\kappa$  is high compared to that of increase in  $\sigma_s$ , the overall trend of  $\beta^*$  shows decreasing trend. As pore density increases, the size of the unit cell decreases but the numbers of unit cells participate in radiative heat transfer increases. As a result, the surface area of solid responsible for absorption and scattering of the radiation also increases. This ultimately results into increase of  $\beta^*$  with PPC values.

The variation of scattering albedo ( $\omega^*$ ) with solid reflectivity as a function of pore density is shown in Fig. 5 (b). As  $\rho_s$  increases, the scattering coefficient ( $\sigma_s$ ) increases and extinction coefficient ( $\beta^*$ ) decreases. These result into increasing nature of  $\omega^*(= \sigma_s / \beta^*)$  with  $\rho_s$  values. Due to isotropic scattering assumption, for a given solid reflectivity, the  $\omega^*$  is found to be insensitive to the pore density values. Similar trend of  $\beta^*$  and  $\omega^*$  for cubic cell porous structure were observed in the work of Fu et al. [1997]. The difference in the results of cubic cell [Fu et al. 1997] and present tetrakaidehedra cell structure is due to different SA by V ratio of both the cell structures.

## CONCLUSION

A combined analytical-numerical model is developed to determine radiative properties of an open cell foam structure that is represented by tetrakaidehedra unit cell. The results obtained using the present model show that both effective reflectance and transmittance of the unit cell increase with increasing reflectivity of the solid surface ( $\rho_s$ ). Similar trend was observed for a cubic unit cell. However, due to high surface area to strut volume ratio of the tetrakaidehedra structure, its radiative properties are more sensitive to  $\rho_s$  values. With increasing  $\rho_s$ , the extinction coefficient shows decreasing trend while the scattering albedo ( $\omega$ ) shows increasing trend. As expected, extinction coefficient of the porous media

increases with the pore density. However, for a given  $\rho_s$ , the scattering albedo remains same for any PPC values.

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### **KEYWORDS**

Tetraikadehedra structure, Open cell foam, Radiative properties, Unit cell, Zonal method