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AN ANALYTIC EXPRESSION FOR RADIATION VIEW FACTORS BETWEEN TWO PLANAR TRIANGLES WITH ARBITRARY ORIENTATIONS

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INTRODUCTION

Radiation view factors, also known as shape factors or configuration factors or form factors, play a key role in the computation of radiation heat transfer (Howell et al., 2011, Modest, 2013) as well as global illumination in computer graphics (Schröder and Hanrahan, 1993). The view factors between two surfaces is defined as the fraction of diffuse radiation from one surface that is intercepted by the second surface.

Unless the configuration of surfaces is such that analytic expressions are available in literature (Howell, 2010), view factors have to be computed numerically. The most general method, though computationally expensive, is the Monte Carlo method (Mazumder and Ravishankar, 2012). Among deterministic methods, the so-called contour integral method uses Stokes' theorem of vector calculus to reduce a double area integral into a double line integral along the edges of the participating surfaces, thereby reducing the computational load (Howell et al., 2011, Modest, 2013). The resultant double line integral can be evaluated by any numerical quadrature scheme.

When faced with the necessity to calculate view factors numerically, we chose another deterministic method that is general enough – Nusselt's unit sphere method (Howell et al., 2011, Lipps, 1983). Our primary reason for gravitating towards the unit sphere method over the contour integral method was simply our liking for the geometric appeal and interpretation of the unit sphere method. In addition, our rationale was that the unit sphere method also succeeded in reducing a 4 dimensional integral to a 2 dimensional integral, though the resultant integral is an area integral on one of the two surfaces.

One of the questions we could have tried answering is whether one of the two methods, unit sphere or contour integral, is better suited for numerical calculations, though the gravitation of the radiation heat transfer community towards the contour integral method seems to have answered it. Instead, we wanted to see if the unit sphere method could give us expressions that reduce the computation from an area integral to a 1D integral, or even better, an analytic result. Through much of our work, we were not aware of the work of Schroder and Hanrahan (Schröder and Hanrahan, 1993) who had answered the question in the affirmative: it was indeed possible to find analytic expressions for view factors between a limited class of surfaces – plane polygons, irrespective of their orientations. This is not an excessive restriction since most arbitrary surfaces are discretized into, usually, planar triangular facets by mesh generation software.

We arrive at a result similar to theirs, though the path taken is quite different. For one, Schroder and Hanrahan use the contour integral method as the starting point, We use the unit sphere method. Unlike their results, which can appear quite complicated because of the presence of complex square roots, the sign of which have to be chosen carefully, we have tried, and succeeded in our assessment, to render the expressions for view factors in a form more suitable for use in numerical codes. One key way we have tried to differentiate our work from that of Schroder and Hanrahan (Schröder and Hanrahan, 1993) is to render the expressions for view factors in a symmetric form so that it becomes apparent that the reciprocity of view factors between two surfaces is obeyed. Like Schroder and Hanrahan, our final expression also contains the dilogarithm function (Olver et al., 2010). As Mazumder and Ravishankar (Mazumder and Ravishankar, 2012) suggest, the analytic nature of the final result presented here lies in the eyes of the reader. In our view, since the dilogarithm function can be computed using different power series expressions (Osácar et al., 1995, Ginsberg and Zaborowski, 1975), we claim it qualifies as an analytic result.

To summarize our work, we have derived analytical expressions for the view factors between two planar triangles with arbitrary orientations. Unlike the works of Schroder and Hanrahan (Schröder and Hanrahan, 1993), the analytical expressions derived here preserve the expected symmetry of view factors. It is straightforward to see, even without numerical computation, that the expressions derived here satisfy reciprocity. Crucially, no numerical quadrature is necessary for this work. It is possible that this method can become the method of choice for computing view factors in radiative transfer simulation software.

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